Excerpt from the Proceedings of the 2011 COMSOL Conference in Boston

Coupled Electromagnetic – Dynamic FEM Simulation of a High Frequency MEMS Energy Harvester

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Abstract: In this study, a detailed finite element model coupling the motion dynamics and electromagnetics of a diaphragm based MEMS vibration energy harvester is presented. The energy harvester converts low frequency vibrations to high frequency response by magnetic actuation of a diaphragm carrying coils. AC/DC electromagnetics, solid mechanics and moving mesh modules are coupled together in one 3-D model to analyze the performance of the MEMS vibration energy harvester. With an input vibration at 120 Hz, the diaphragm carrying the coils resonates at 2200 Hz. With 5 Ω coil resistance, a peak-to-peak voltage of 7.2 mV and a peak electrical power of 1.45 µW can be generated from the electromagnetic device having a volume of roughly 0.25 cm^3.

Keywords: vibration energy harvesting, electromagnetics, MEMS, energy scavenging, coupled physics

1. Introduction

MEMS vibration energy harvesting has been the hot topic in recent years. As the battery technology improves slowly and there are dimension considerations, harnessing energy from vibration has been one solution to provide electrical energy. MEMS vibration energy harvesters utilize electromagnetic, piezoelectric or electrostatic energy conversion to convert ambient vibrations into electrical energy.

In electromagnetic harvesters, magnet(s) and a coil moving relative to each other creates an electrical current through the coil due to electromagnetic induction. In this MEMS energy harvester, this relative motion between the coils and the magnet is created by ambient vibrations.

2. Theory for Electromagnetic Energy Harvesting

In electromagnetic energy harvesters, there is a magnet and coil moving relative to each other. The voltage generated is the rate of change of magnetic flux in the coil area and denoted by (1).

$$V = -\frac{d\Phi}{dt} = -\frac{d\left(\sum_{i=1}^{n}\left(\vec{B}\cdot\vec{A_i}\right)\right)}{dt} \tag{1}$$

Here Φ is the magnetic flux density over the coil area, $\vec{B}$ denotes the magnetic flux vector, $\vec{A_i}$ is the coil area where electromagnetic induction occurs, and t is time. (1) can be expanded to give a more clear formulation relating the magnetic flux to voltage (2).

$$V = -B\sum_{i=1}^{n} A_i - \sum_{i=1}^{n} A_i \frac{dB}{dt} \tag{2}$$

The rate of change of magnetic flux over the area is extracted from Comsol Multiphysics postprocessing, and used to calculate the transient voltage output.

The electrical power obtained with an electrical load connected is then denoted by (3).

$$P_{el} = \frac{V^2}{(R_L + R_c)^2}\cdot R_L \tag{3}$$

More detailed analysis on the physics and optimization of electromagnetic energy harvesters can be found in [1], [2].

3. Coupled physics simulation of the MEMS Electromagnetic Energy Harvester using COMSOL Multiphysics

Our system consists of an NdFeB magnet attached on a flexible Parylene diaphragm and another diaphragm below that carries the coils on it. The upper diaphragm with the mass has a relatively low natural frequency 120 Hz and the lower diaphragm has a resonance frequency of 2000 Hz. The lower diaphragm carries coils and Ni layer in the middle for electromagnetic actuation. With this configuration, as the magnet approaches the lower diaphragm it catches and bends the lower diaphragm upwards. As the upper diaphragm moves upward, it releases the lower diaphragm and natural resonance for the lower diaphragm occurs. By this method, the low frequency vibrations are converted to high frequency response, increasing the electrical power.

As it can be concluded from (1)-(3), the key parameters for performance calculation of electromagnetic energy harvesters is the rate of change of magnetic flux, the displacement and velocity of the magnet and diaphragm. For electrical flux calculations, the changing magnetic field and flux on the coil area must be calculated accurately.

In Comsol Multiphysics, we coupled the Solid Mechanics Module (transient), and AC/DC Electromagnetics Module with a moving mesh to simulate the complete behavior of the system. The magnetic field lines obtained from the model for this system is seen in Figure 1.

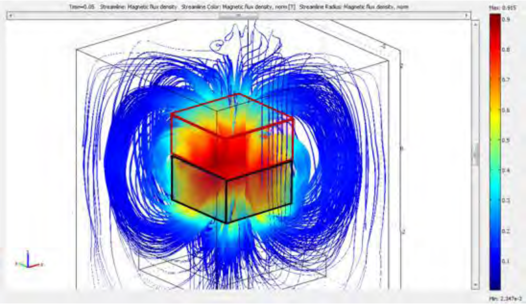

Figure 1. The model coupling Electromagnetics and Solid Mechanics. Magnetic flux density is shown.

The magnetic flux on the coil boundary (1) is calculated by boundary integration over the coil area and shown in Figure 2.

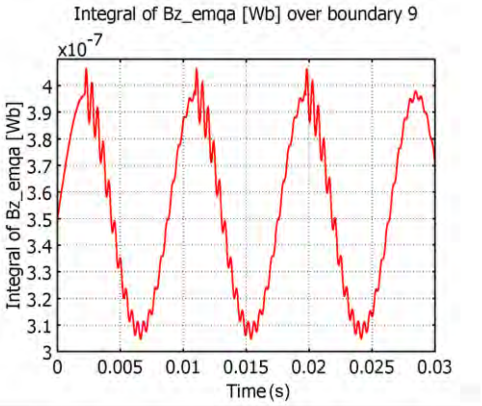

Figure 2. Boundary integration of magnetic flux in z-direction over the coil turn with the largest area.

In Figure 2, the signal having a larger magnitude and a lower frequency is the magnetic flux change caused by the motion of the magnet. The small amplitude and high frequency signal is the magnetic flux change due to motion of the actuated diaphragm. The diaphragm motion creates a relatively lower amount of change in the magnetic flux density, because the displacement amplitude of the diaphragm is in the range of micrometers.

Once the velocity vectors are obtained for both magnet and diaphragm, the voltage generated on the coil is calculated. The voltage is calculated combining the electromagnetic flux values with the velocity vectors in MATLAB. Voltage generated from MEMS electromagnetic energy harvester is shown in Figure 3.

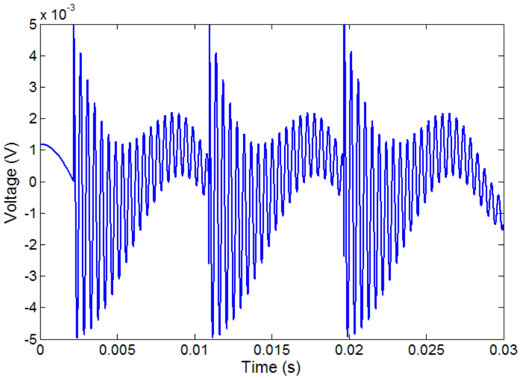

Figure 3. The voltage obtained from vibration energy harvester. The peak voltage occurs where the actuation is initiated.

Figure 3 shows that the maximum voltage is generated at the locations where the high frequency signal is seen in Figure 2. Here, the diaphragm motion occurs at a high frequency, which means the derivative of the magnetic flux is much higher than the low frequency signal. So, it is natural to see higher voltages where the diaphragm motion occurs. To form the relation more clearly, the voltage generated by an electromagnetic energy harvester can also be expressed by (4).

$$V = \frac{B l_e \omega \left(\frac{\omega}{\omega_n}\right)^2 Y \cdot \cos(\omega t + \varphi)}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\zeta \frac{\omega}{\omega_n}\right)^2}} \qquad (4)$$

Where B is the magnetic field density, l_e is the effective coil length, ω is the resonance frequency, ω_n is the natural frequency of the structure, Y is the input displacement amplitude, φ is the phase of excitation, and ζ is the total damping ratio and equal to the summation of mechanical and electrical damping ratio $\zeta_m + \zeta_e$. Here Bl_e term includes $\omega^{1/2}$ component, so the voltage generated is proportional to $\omega^{3/2}$. Thus, the high amplitude & high frequency signal in Figure 3 can be explained by the diaphragm motion.

Analyzing Figure 3 in more detail, the decaying voltage response is parallel with the decaying vibration motion. The decay rate is calculated from the damping of the diaphragm. The damping ratio of 0.8 cm x 0.8 cm diaphragm is calculated using (5) [3].

$$b_m = \frac{\left(3\pi\mu b + 0.75\pi b^2\sqrt{2\rho_a\mu\omega}\right)m_{eq}}{\rho_b b^2 h} + \frac{\mu b^2 m_{eq}}{\rho_b g_0^3 h} + \frac{\eta}{\omega}k_{eq} + \frac{(0.23h^3)}{L^3}2m_{eq}\omega_n \qquad (5)$$

Here, μ : dynamic viscosity of air, Pa.s
b : width of the diaphragm, m
ρ_a : density of air, kg/m^3
ω : oscillation frequency of the diaphragm
ρ_b : density of diaphragm material, kg/m^3
h : thickness of the diaphragm, m
m_{eq} : equivalent mass of diaphragm, kg
g_0: distance between the diaphragm and nearby rigid wall, m
η : structural damping coefficient
k_{eq} : equivalent stiffness of the diaphragm, N/m
L : length of the diaphragm, m
ω_n : natural frequency of the diaphragm

The maximum electrical power extracted from an electromagnetic energy harvester is when the load resistance is equal to the coil resistance, and calculated by (6).

$$P = \frac{V^2}{R_c + R_l} = \left(\frac{R_l}{R_c + R_l}\right)\frac{m_{eq}\xi_e\left(\frac{\omega}{\omega_n}\right)^3\omega^3 Y^2}{\left(1-\left(\frac{\omega}{\omega_n}\right)^2\right)^2 + \left(2\zeta\frac{\omega}{\omega_n}\right)^2}\cos^2(\omega t + \varphi) \qquad (6)$$

Here, R_l is the load resistance, R_c is the coil resistance, m_{eq} is the equivalent mass, ξ_e is the electrical damping ratio and ζ is the total damping ratio. As it can be concluded from Equation (6), the power generated is proportional to the cube of the excitation frequency. The electrical power calculated is shown in Figure 4.

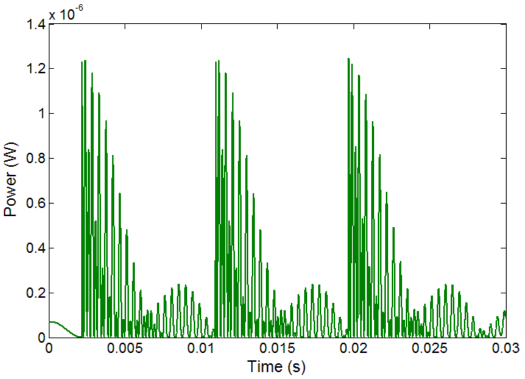

Figure 4. The electrical power generated by the diaphragm based electromagnetic harvester. The maximum power occurs at the actuation instant, where diaphragm displacement is maximum.

4. Details of the Modeling

Coupling physics in a model is mostly problematic especially for dynamic problems and sometimes leads to non-converging solutions if the problem is tried to be solved at a single step. In this model, to get an initial value of the magnetic flux, the elements in solid mechanics module (diaphragm and magnet) and moving mesh motion (u,v,w) are kept fixed. The magnetic flux in the coil area in a non-moving system is shown in Figure 5.

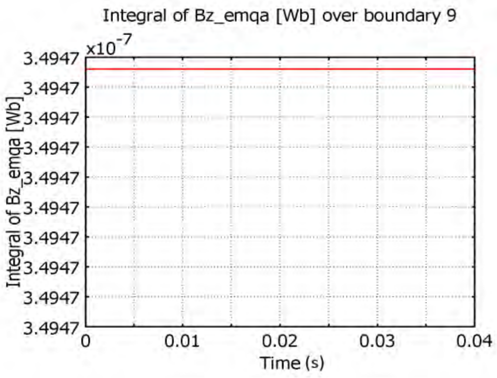

Figure 5. The magnetic flux on the coil area when all elements are fixed.

After the first solution is obtained, the solid mechanics motion simulating vibration and moving mesh motion (u,v,w) are included in the model. The equation specifying the vibration motion of the lower high frequency diaphragm is shown in (7).

$$X_1 = A_1 \cdot e^{-\zeta \cdot w_n \cdot t} \cdot \sin(\omega_{n1} \cdot t) \tag{7}$$

Here A denotes the initial displacement, ζ is the damping ratio in air, w_n is the natural frequency and t is the time.

For the magnet attached to diaphragm above, constant sinusoidal motion is assumed. The equation of the diaphragm motion is shown in (8).

$$X_2 = A_2 \cdot \sin(\omega_{n2} \cdot t) \tag{8}$$

The motion of the high frequency diaphragm and magnet are plotted in Figure 6.

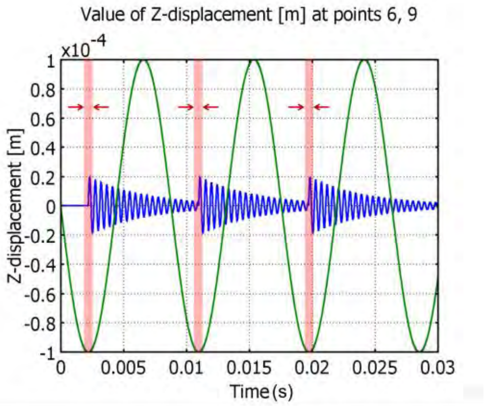

Figure 6. The displacement of the high frequency diaphragm and magnet.

In Figure 6, the diaphragm motion starts when the magnet is at its lowest z-position. The red highlighted regions denote the catch and release instances of the diaphragm (the minimum points of the green line where t=2.19 ms, t=10.96 ms, and t=19.8 ms). Here the magnet actuates upward motion in the diaphragm.

5. Conclusions

A coupled physics model for a vibration energy harvester is presented. This model combines AC/DC Electromagnetics, Solid Mechanics and Moving Mesh modules together to give realistic results for the voltage output of a novel MEMS energy harvester. In this model, the varying magnetic flux profile is calculated integrating the moving mesh module. Thus apart from simplifying assumptions, a detailed finite element model of the electromagnetic and dynamics coupled system is introduced. This approach can simplify the optimization for magnetic fields for maximum performance of the MEMS energy harvester.

6. References

[1] C. Cepnik, O. Radler, S. Rosenbaum, T. Ströhla, and U. Wallrabe, "Effective optimization of electromagnetic energy harvesters through direct computation of the electromagnetic coupling," *Sensors and Actuators A: Physical*, vol. 167, no. 2, pp. 416-421, Jun. 2011.

[2] E. Bouendeu, A. Greiner, P. J. Smith, and J. G. Korvink, "Design Synthesis of Electromagnetic Vibration-Driven Energy Generators Using a Variational Formulation," *Journal of Microelectromechanical Systems*, vol. 20, no. 2, pp. 466-475, Apr. 2011.

[3] H. Hosaka, K. Itao, and S. Kuroda, "Evaluation of energy dissipation mechanisms in vibrationalmicroactuators," in *IEEE Workshop on Micro Electro Mechanical Systems, 1994, MEMS '94, Proceedings*, 1994, pp. 193-198.