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Three Dimensional Numerical Study of the Interaction of Turbulent Liquid Metal Flow with an External Magnetic Field

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Abstract: In this paper, we present the numerical study of the interaction of wall bounded turbulent liquid metal flow with the magnetic field of a permanent magnet. The study is motivated by Lorentz Force Velocimetry (LFV), a technique used for non-contact flow measurement. The numerical analysis for this work is performed using the commercial finite element solver Comsol™ Multiphysics. The total Lorentz force acting on the magnet system is evaluated at different flow Reynolds numbers and at different magnet distances from the duct. The obtained results help in understanding the dynamics of liquid metal flows exposed to inhomogeneous magnetic fields. The study could also help in benchmarking Comsol™ Multiphysics for liquid metal based magnetohydrodynamic flows.

Keywords: Lorentz force velocimetry, Magnetohydrodynamics, Turbulence modelling.

1. Introduction

Contact measurement of flow rates during metallurgical processes is often a difficult task owing to the high temperatures of the metal melts. Therefore, over the last few years a lot of innovative non-contact flow measurement techniques have been proposed [1,2]. Lorentz Force Velocimetry (LFV) is one such technique used to determine flow rates in electrically conducting fluids [3,4]. Such fluids when exposed to external magnetic fields produce Lorentz forces by virtue of eddy currents induced in them. This Lorentz force brakes the fluid flow according to Lenz's law. Also, an equal and opposite force (Kelvin force) acts on the magnet system according to Newton's third law. LFV works on the principle of measurement this Kelvin force acting on the magnet system using force sensors. Typically, for LFV applications real and complex magnet systems with inhomogeneous magnetic fields are used.

Therefore, in this paper, we place emphasis upon understanding the interaction of liquid metal flows with inhomogeneous magnetic fields. We consider the fundamental problem of a liquid metal flow exposed to a soft-iron yoke based permanent magnet. The liquid metal flows in straight square duct (Hc x Dc x Wc) with electrically insulated walls (Figure 1).

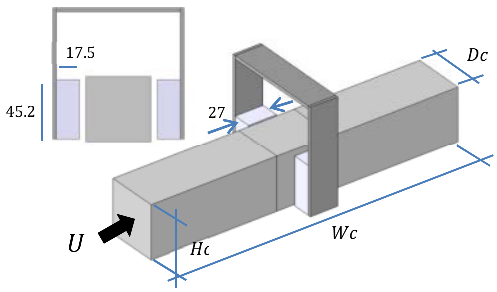

Fig.1: Schematic of the problem with geometry parameters in mm.

It is well known from simple scaling estimates that the Lorentz force , f, acting on the magnet system is $f \sim \sigma U B^2$ where, σ is the electrical conductivity of the fluid, U is the magnitude of the velocity based on flow rate and B is the magnitude of the magnetic field acting inside the fluid [5]. There is a linear dependence between the flow rate and the measured Lorentz force. Therefore, by properly calibrating the flow meter it is possible to measure the fluid velocity. Furthermore, since the force is proportional to the square of the magnetic field, which decays with increasing the distance of the magnet from the duct, it is possible to understand and improve the sensitivity of the flow meter by providing some scaling estimated for the decay of Lorentz force. Hence, the main goal of this paper is the numerical evaluation of the total Lorentz force and the spatial distribution of Lorentz force density in the fluid. Parametric studies are performed to evaluate the variation of Lorentz force with Reynolds number of the flow and with distance of the magnet system from the duct. The effect of Lorentz force distribution on the flow dynamics is also analysed. The analyses provide good reference results for the numerical calibration of Lorentz force flowmeters.

The paper is organised as follows: in the next section we provide a brief description of the problem setup and the numerical methodology adapted in Comsol™ Multiphysics including all the governing equations. Section 3 describes the obtained numerical results and their subsequent discussion. Finally, conclusions and future research plans are presented in section 4.

2. Problem description and numerical methodology

We consider an electrically conducting non-magnetic liquid metal flow in a square duct of cross section area 50 mm x 50 mm and length 350 mm. In order to provide numerical data for later experimental work, the liquid considered is the eutectic alloy $Ga_{68}In_{20}Sn_{12}$, which is a liquid at room temperature. The fluid is incompressible with an electrical conductivity of $3.3 \cdot 10^6 (1/\Omega\text{m})$ and density 6360 (Kg/m^3). The flow is exposed to the magnetic field of a soft-iron yoke permanent magnet with width 27 mm, height 45.2 mm and depth 17.5 mm. These values were chosen based on shape optimisation analysis. The magnetic field of the permanent magnet is defined using remanent induction strength of 1.3 Tesla magnetised in the spanwise direction.

For this problem setup, the Navier-Stokes equations along with the equations for magnetic vector potential and electric scalar potential are solved using the commercial finite element solver Comsol™ Multiphysics (version 4.2) through a coupling between the AC/DC and CFD application modes. K-ε based turbulent model is used for the fluid dynamic calculation with the Lorentz force included as a volume source in the momentum equation. The fluid velocity is then supplied to the AC/DC module for the magnetostatic calculation.

The governing equations used in AC/DC magnetostatic module are as follows [6]:

$$\nabla \times (\mu_0^{-1}\mu_r^{-1}(\nabla \times \boldsymbol{A}) - \mu_0^{-1}\boldsymbol{B}_r) - \sigma\boldsymbol{u} \times (\nabla \times \boldsymbol{A}) + \sigma\nabla V = 0 \quad (1)$$

$$\nabla \cdot (\sigma\boldsymbol{u} \times (\nabla \times \boldsymbol{A}) - \sigma\nabla V) = 0 \quad (2)$$

Here, $\boldsymbol{A}$ is the magnetic vector potential, V is the electric scalar potential, σ is the electrical conductivity, $\boldsymbol{u}$ is the velocity, $\boldsymbol{B_r}$ is the remanence of permanent magnets, μ_0 is the magnetic permeability in the air, and μ_r is the relative magnetic permeability. The first equation is Ampere's law and the second equation is the current conservation. These two equations are solved in each subdomain of the FEM model. Some terms vanish depending on subdomain physics. For example, terms $\mu_0^{-1}\boldsymbol{B_r}$ and $\sigma\boldsymbol{u} \times (\nabla \times \boldsymbol{A})$ are used only in permanent magnet and liquid metal subdomain, respectively.

To solve the system of equations (1)-(2) boundary conditions are used. For the internal boundaries of the model so-called continuity conditions are ensured:

$$n \times (H_1 - H_2) = 0; \quad n \cdot (J_1 - J_2) = 0 \quad (3)$$

For the external boundaries of the model so-called magnetic insulation condition is imposed:

$$n \times \boldsymbol{A} = 0; \quad \boldsymbol{V} = 0 \quad (4)$$

The fluid flow in the duct is modelled by Reynolds averaged Navier-Stokes based (RANS) k-epsilon turbulence model:

$$\rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = \nabla \cdot \left[-p\boldsymbol{I} + (\mu + \mu_T)(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T) - \tfrac{2}{3}\rho k\boldsymbol{I}\right] + \boldsymbol{F} \quad (5)$$

$$\nabla \cdot \boldsymbol{u} = \boldsymbol{0} \quad (6)$$

$$\rho(\boldsymbol{u} \cdot \nabla)k = \nabla \cdot \left[\left(\mu + \frac{\mu_T}{\sigma_k}\right)\nabla k\right] + P_k - \rho\varepsilon \quad (7)$$

$$\rho(\boldsymbol{u} \cdot \nabla)\varepsilon = \nabla \cdot \left[\left(\mu + \frac{\mu_T}{\sigma_\varepsilon}\right)\nabla\varepsilon\right] + C_{\varepsilon 1}\frac{\varepsilon}{k}P_k - C_{\varepsilon 2}\rho\frac{\varepsilon^2}{k} \quad (8)$$

Here, $\mu_T = \rho C_\mu \frac{k^2}{\varepsilon}$ is the turbulent viscosity, $P_k = \mu_T[\nabla\boldsymbol{u}: (\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T)]$ is the turbulence production, p is pressure, k is the turbulent kinetic energy, ε is the dissipation of k, σ_k, σ_ε, C_μ, $C_{\varepsilon 1}$ and $C_{\varepsilon 2}$ are constants of turbulence model, $\boldsymbol{F}$ is the volumetric Lorentz force which is calculated in AC/DC module, $\boldsymbol{u}$ is the velocity field. Magnetostatic and CFD modules are coupled by $\boldsymbol{F}$ and $\boldsymbol{u}$ terms. In other words, $\boldsymbol{F}$ field is calculated by magnetostatic module using $\boldsymbol{u}$ field from CFD module. And $\boldsymbol{u}$ field is calculated by CFD module using $\boldsymbol{F}$ field from magnetostatic module.

The mesh for all the calculations is generated using tetrahedral elements of second order. The mesh sizes were chosen after preliminary calculations to achieve sufficient accuracy and reasonable computational time. The liquid metal region is meshed non-uniformly with a high mesh density in the vicinity of permanent magnets and near the channel walls, because of high magnetic field and velocity gradients (figure 2). The mesh results approximately in 3.5 million degrees of freedom.

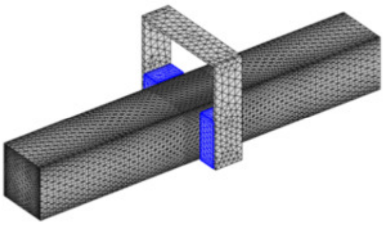

Fig.2: Subdomains of the problem after the mesh generation (air region is not shown).

3. Results and discussion

From the results (figure 3) it is apparent that the maximum magnetic field intensity is near the side walls of the duct but decays rapidly towards to the middle.

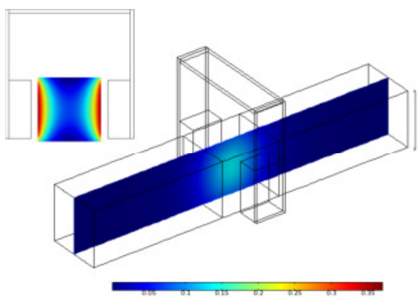

Fig.3: Magnetic field (Tesla) distribution with the magnet at a distance of 5 mm from the duct walls.

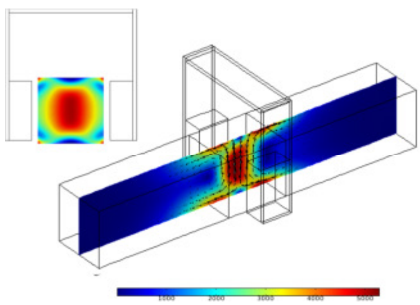

Fig.4: Eddy currents (A/m^2) distribution with the magnet at a distance of 5 mm from the duct walls and Reynolds number 2000.

The interaction of this magnetic field with the fluid flow produces eddy currents which are predominantly in the plane parallel to the fluid flow (figure 4). The interaction between these eddy currents and the magnetic field produce Lorentz forces in the fluid (figure 5). The Lorentz force acts predominantly as a braking force on the fluid. But in regions close to the top and bottom walls of the duct, the Lorentz force accelerates the fluid flow. This is attributed to the fact the eddy currents change direction in this region to complete the current loop (figure 6 and 7).

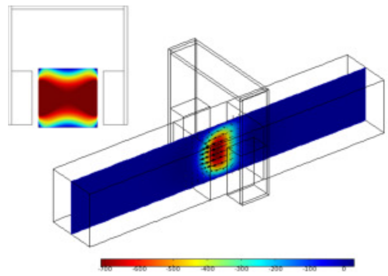

Fig.5: Lorentz force density (N/m^3) distribution with the magnet at a distance of 5 mm from the duct walls and Reynolds number 2000.

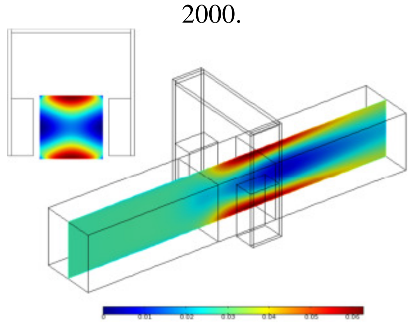

Fig.6: Streamwise velocity (m/s) in the duct with the magnet at a distance of 5 mm from the duct walls and Reynolds number 2000.

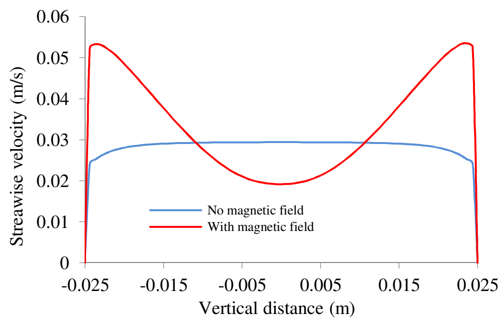

Fig.7: Streamwise velocity (m/s) in the vertical direction at middle of the duct with the magnet at a distance of 5 mm from the duct walls and Reynolds number 2000.

We consider now the influence of magnet position on the total Lorentz force. As expected, the force decays monotonically due to the decrease in the magnetic flux density with distance (figure 8). It is further observed that the force decrease is consistent with an exponential decay.

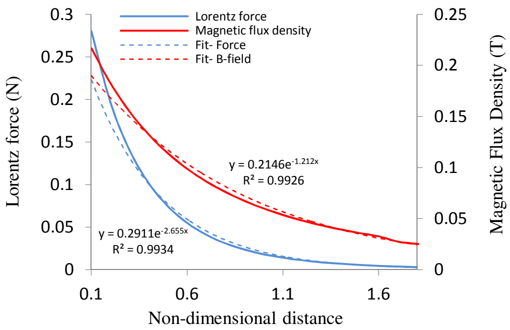

Fig.8: Variation of Lorentz force and area averaged magnetic flux density with distance of the magnet system (non-dimensionalised with half duct width of 25 mm) from the duct at Reynolds number 2000

Further parametric studies are performed to understand the variation of flow Reynolds number on the Lorentz force. Predominantly there is a linear dependence of Lorentz force on flow Reynolds number (figure 9). But for very high Reynolds numbers (> 0.5 million), the force tends towards a non-linearity.

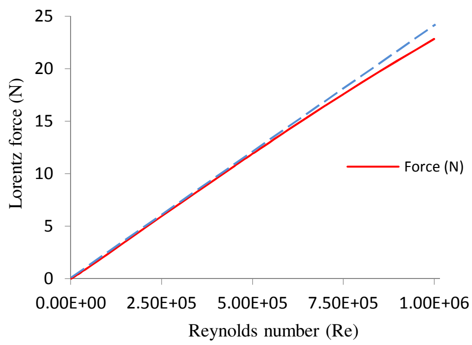

Fig.9: Variation of Lorentz force with Reynolds number with the magnet located at 5mm from the duct walls.

This phenomenon can be attributed to the fact at such high velocities, the induced magnetic field due to the eddy currents in the liquid are comparable to the primary imposed magnetic field of the permanent magnet. In MHD flows, such an effect is often characterised by a non-dimensional parameter known as magnetic Reynolds number (R_m). This parameter can be vaguely interpreted as the ratio of induced to the imposed magnetic field. At high R_m, the magnetic field lines are stretched by the fluid flow thereby reducing the penetration depth of the field lines and resulting in a saturation of the total Lorentz force.

4. Conclusion

Comsol™ Multiphysics was used to numerical study the interaction of liquid metal flow exposed to external magnetic field. The solver was able to reproduce the braking effect on the fluid flow due the Lorentz forces. Also, the regions close to the duct wall where the Lorentz force accelerated the fluid flow was clearly illustrated.

The parametric studies showed an exponential decay of the Lorentz force with increasing distance of the magnet system from the duct. The well-known linear relationship between flow Reynolds number and the total Lorentz force was presented. Furthermore, the force saturation at high Reynolds number due to distortion magnetic field lines by the fluid flow was observed.

5. References

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6. Acknowledgements

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