Simulation of the Spread of Epidemic Disease Using Persistent Surveillance Data

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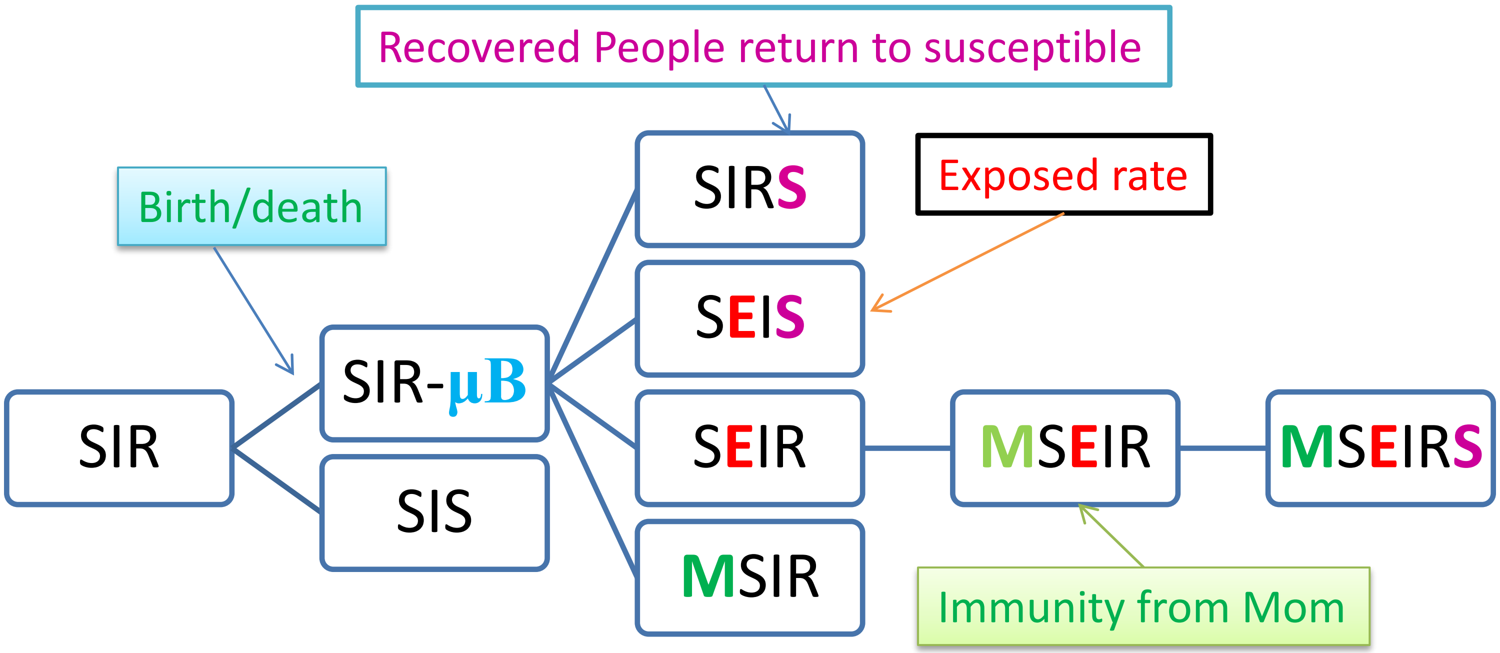
Agenda

- Significance of Mathematical Modeling in Epidemic Disease.
- SIR (Susceptible-Infected-Recovered) --family models and their shortcomings.
- Principle of SIR-HT (Heat Transfer) model, which is proposed as an innovative method in this field.
- Mathematical Description of SIR-HT Model
- Simulation Using COMSOL 3.5a
- Conclusion and Future Work

Significance of Mathematical Modeling in Epidemic Disease

- Explore the **transmission mechanism** of epidemic diseases;
- Obtain insight into potential **cost and outcomes** of the breakout of the disease;
- Evaluate the effectiveness of prevention / control strategies such as immunization and segregation .

Existing Epidemic Models (Deterministic)



S: susceptible; **I**: infected; **R**: recovered;
μ: death; B: birth; **M**: immunity from mother;
E: exposed rate in latent period

SIR Model – Mathematical Description

Define ***s*(*t*)**, ***i*(*t*)** and ***r*(*t*)** be the proportion of the number of susceptible, infected and recovered individuals at time ***t***

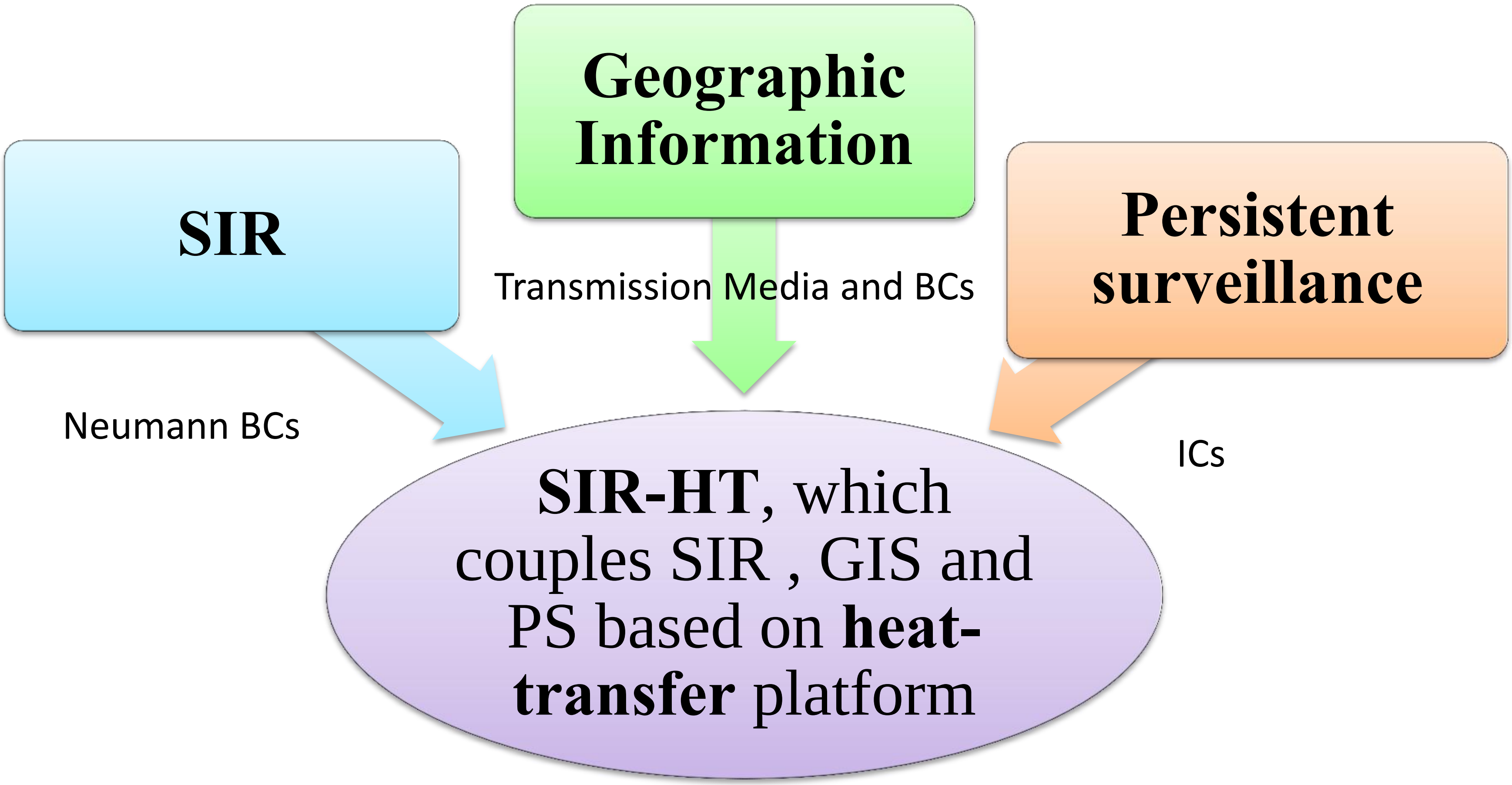
$$
\begin{cases}
\dfrac{ds(t)}{dt} = -\beta\, s(t)\, i(t) \\
\dfrac{di(t)}{dt} = \beta\, s(t)\, i(t) - \gamma\, i(t) \\
\dfrac{dr(t)}{dt} = \gamma\, i(t) \\
s(t) + i(t) + r(t) = 1
\end{cases}
$$

scalar ***β*** is contact rate; scalar ***γ*** is the mean recovery rate

Short-Comings of Existing Epidemic Model

- In an **isolated** community: no interaction between neighboring communities.
- No **spatial variable** such as distance, location, route of transmission , etc.
- No transmission **media**.

Derivation of SIR-HT Model



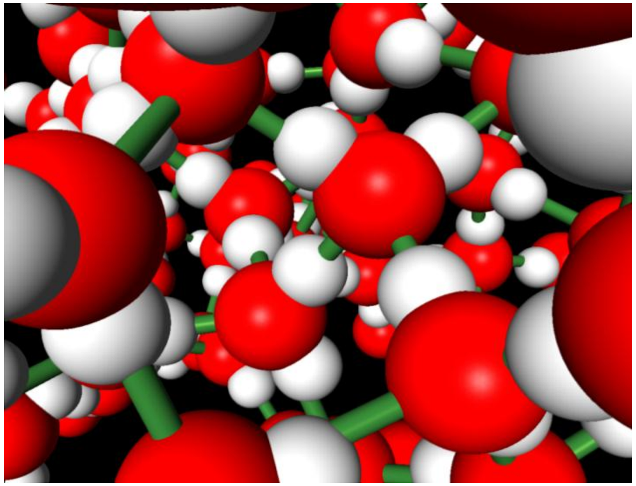
Principle of SIR-HT: Similarity in Diffusion Mechanism of Disease and Heat

Spread of Epidemic Disease

- Contact infection
- Personnel's movement

Diffusion of Heat Energy

- Vibration of atom influences neighboring atoms
- Free electrons carry energy



Principle of SIR-HT:SIR-HT vs. Standard Heat-transfer

SIR-HT model	**Counterpart in Heat transfer model**
Fraction of infective population	temperature (*T*)
Change rate of infective population	Heat-flux(*Q*)
Personnel exchange between neighboring community	Conductivity (*k*)
Road (including local, high-way, free-way)	**Thin but highly conductive layer**
Terrain conditions (lake, mountain, etc)	Boundary conditions
Persistent surveillance data	Initial conditions
Conservation of infective	Law of conservation of energy
The transmission of infectious between neighboring communities is proportional to the difference of their infective rate	Fourier law

Mathematical Description of SIR-HT: Governing Equations

$$\begin{cases} \rho C_p \dfrac{\partial i(X,t)}{\partial t} + \nabla \cdot (-K(X)\nabla i(X,t)) = Q_{\inf} - Q_{rec} \\ Q_{rec} = \gamma(X) i(X,t) \\ Q_{\inf} = \beta(X) s(X,t) i(X,t) \end{cases}$$

β(X) is location-related **contact infection tensor**;
γ(X) is **recovery rate**;
ρ is population density;
C_p is a time-scaling coefficient (dimensionless);
Q_{int} is the incremental infective caused by contact infection;
Q_{rec} is the decremented infective caused by recovery;
$\nabla \cdot (-K(X)\nabla i(X,t))$ indicates the infective change caused by inter-community personnel exchanging.

Mathematical Description of SIR-HT: BCs Introduced by Road

$$\begin{cases} d_{rd}\rho_{rd}C_{rd} + \nabla\cdot\left(-d_{rd}\kappa_{rd}\nabla i(X,t)\right) = -n\cdot q \\ q = -K(X)\nabla i(X,t) \end{cases}$$

d_{rd} is transportation bandwidth;
ρ_{rd} is the passenger density;
C_{rd} is a coefficient;
the transportation network is translated into boundary condition, denoted as $\partial\Omega_{rd}$
Counter-part term in COMSOL: **thin but highly conductive layer**

Mathematical Description of SIR-HT: Initial Condition and BCs

- **Initial conditions** are derived from **persistent surveillance data**;
- The disease **transmission media** is defined according to geographic information.

Flowchart of SIR-HT Framework

1) Formulate the heat-transfer medium according to geographic information

2) Instill the SIR model into SIR-HT model

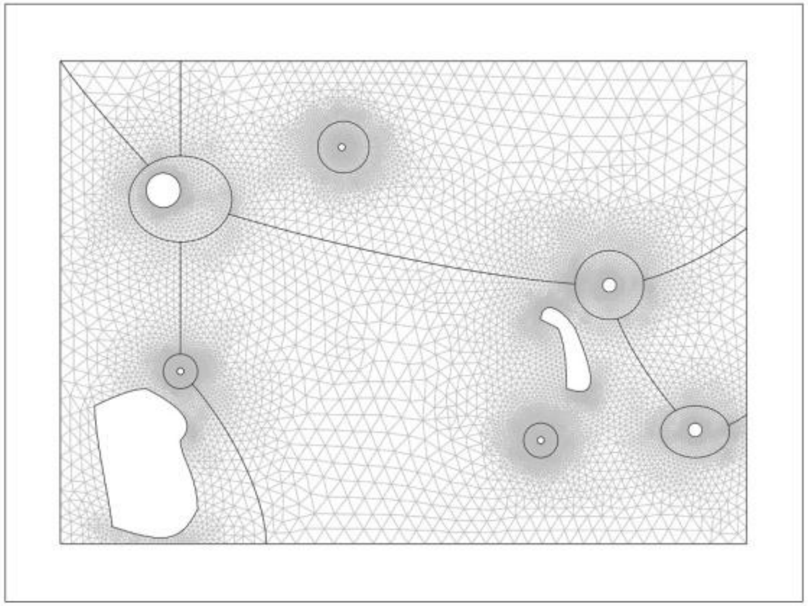
3) Obtain the initial/boundary conditions of the heat transfer problem according to persistent surveillance data

4) Simulate the spread of epidemic disease by solving the transient heat-transfer problem

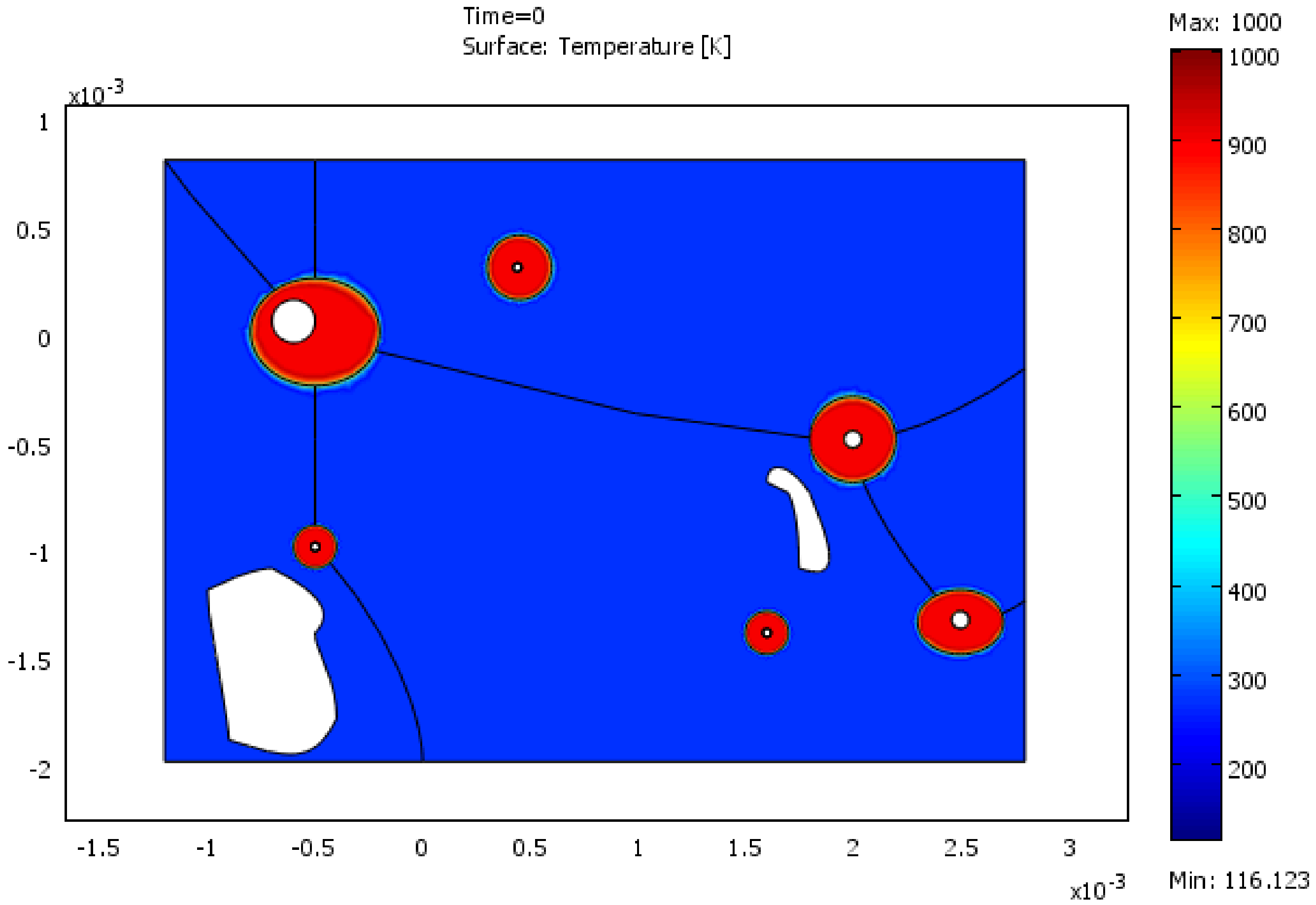
Simulation Experiment: Spread of Flu at a Sample Site Near Minneapolis

Map of a sample site near Minneapolis

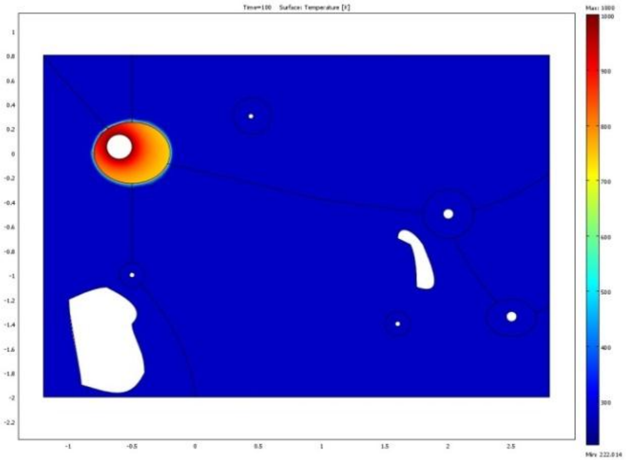
Heat-transfer model derived from sample site



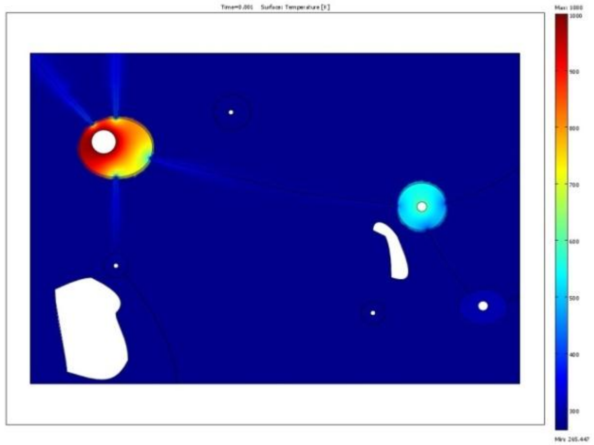
Experiment: Animation of the Spread of Epidemic Flu



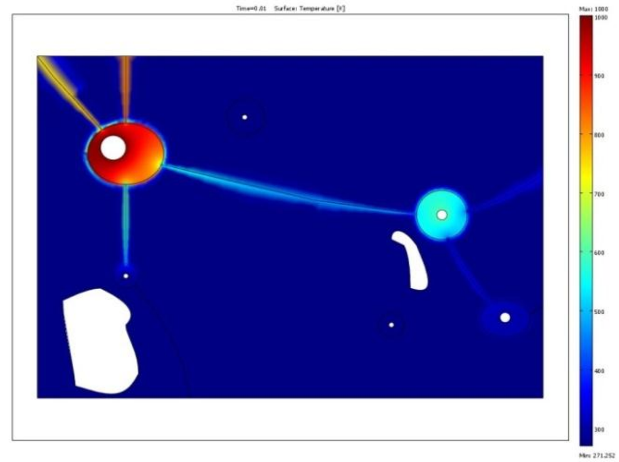
Experiment: Spread of Epidemic Flu with Time



Day 1



Day 3



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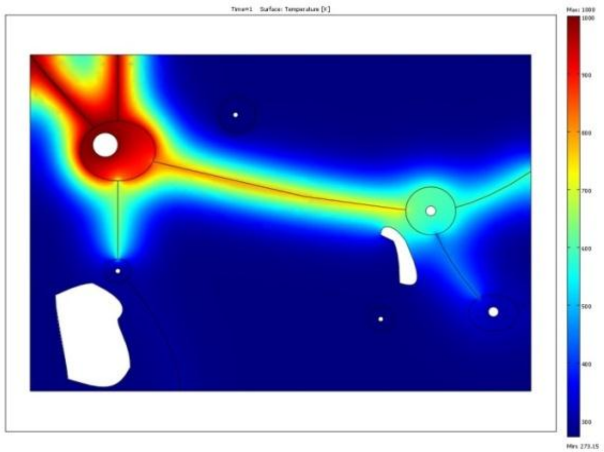

Day 10

Conclusions

- A novel deterministic epidemic model is developed and implemented using COMSOL 3.5a;
- The simulation result shows infectious disease spread within residential area or along transportation network, which is basically consistent with our expectation;
- A more **critical validation** about the SIR-HT model is needed with the support and collaborations of experts in multidisciplinary areas such as medical science, sociology, statistic, optimization, geology science, and public health, etc.

Future Work

- **Validation** of proposed mathematical model;
- Effect of public **prevention strategy** and **medical treatment** over the SIR-HT;
- Introduction probability into SIR-HT model to achieve **stochastic** description about the spread of epidemic disease;
- Effects of **air-line** transportation over SIR-HT;
- **Global** tracking/analysis platform for epidemic disease;
- Promote SIR-HT framework into other applications such as immigration of locust, spread of cancer cells, etc.

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Question and Answer

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