Modelling Microwave Scattering from Rough Sea Ice Surfaces

Xu Xu[1], Anthony P. Doulgeris[1], Frank Melandsø[1], Camilla Brekke[1]
1. Department of Physics and Technology, UiT The Arctic University of Norway, Tromsø, Norway

Abstract: In this paper, COMSOL Multiphysics® was used to simulate the microwave scattering from the rough sea ice surface. A nonperiodic model and a periodic model were built. The nonperiodic model considers the rough surface of finite length and introduces a tapered incident wave. In this model, the strategy of total and scattered-field decomposition (TSFD) was used to formulate the finite-element method (FEM). The computational area was split into a scattered-field region and a total-field region so that the incident wave can be impressed closer to the rough sea ice surface. The periodic model considers the periodic rough surface by introducing Floquet periodic boundary conditions. The incident wave is excited by the port boundary condition so this model is based on the total-field formulation. The two models were tested to simulate the radar cross section (RCS) of scattering from sea ice surfaces at C band (frequency 5.4GHz). The results were compared with the Small Perturbation Method (SPM) and good agreements were achieved.

Keywords: Microwave scattering, rough sea ice surface, radar cross section

1. Introduction

Sea ice is an important indicator for global climatic changes. Reliable and constant monitoring is important for acquiring the state of sea ice. Microwave remote sensing has been widely used to monitor sea ice due to its independence of light and rough weather conditions in remote Arctic area. The radar emits a microwave that interacts with the sea ice and receives the scattered microwave that carries the information of the sea ice. Electromagnetic (EM) Modeling studies can be undertaken to interpret the information. By simulating the scattering from the hypothetical sea ice, we can also find the relation between the received radar signal and sea ice properties, which helps to understand how to retrieve geophysical parameters of sea ice from the radar signal.

Many numerical and analytical methods have been proposed to simulate the microwave scattering from sea ice. The FEM, as a classic numerical method, has been used to study the soil scattering [1, 2]. In this paper, COMSOL Multiphysics® (COMSOL) is used to accomplish the FEM numerical modeling for the sea ice scattering. Based on the type of the simulated sea ice surface, two models are built. One is a nonperiodic model which simulates the surface of finite length. The other model is periodic by introducing Floquet periodic boundary conditions to construct the periodic rough surface. Considering the wavelength of microwave and the size of the rough surface, both models are built using the Radio Frequency (RF) module in COMSOL.

The paper is organized as follows: Section 2 introduces the simulation of sea ice. Section 3 and 4 detail the theory and modelling procedure of the nonperiodic and periodic model respectively. Simulation results are given in Section 5 and conclusions in Section 6.

2. Sea ice simulation

For the geometry of the sea ice, a Gaussian distributed rough surface is generated by using the method in [3]. The rough surface involves two parameters: the root mean square (rms) height and correlation length. For the nonperiodic model, the rough surface should be large enough to account for the statistical property. The surface is created by connecting points that are $\lambda/10$ apart, where λ is the wavelength of the incident wave. For the periodic model, we enforce the endpoints of the surface have the same heights, which ensures the periodic surface boundaries match at the begining and the end. The rough surface is generated in MATLAB® and then imported into COMSOL to build the geometry of the model.

The complex permittivity of sea ice can be calculated by salinity and temperature data with the use of the Polder-van Santen-de Loor mixture model [4].

3. Nonperiodic model

3.1 Theory

For a scattering problem, the finite element solution can be formulated in terms of either the total or the scattered field. The difference between total field and scattered field formulations is how the incident wave is excited. For the total field formulation, the incident wave is excited by a source away from the surface of scatterers. For the scattered field formulation, the incident wave is known and introduced directly on scatterers. Only the scatterred field is solved. In general, the total field formulation is less accurate because it can cause dispersion error with the propagation of incident wave.

As shown in Figure 1, the scatter (sea ice) is the semi-infinite half-space medium in this study. Within the domain of sea ice, there is no meaning to decompose the total field into the incident field and the scattered field. Total field formulation should be conducted to describe the electromagnetic wave inside the sea ice area. The method of total- and scattered-field decomposition (TSFD) is used to solve the scattering from the sea ice. In the air domain, the scattered field formulation is applied. The wave equation to be solved is

$$\nabla\times\mu_{r1}^{-1}\left(\nabla\times\vec{E}_1\right)-k_0^2\epsilon_{r1}\vec{E}_1=0 \qquad (1)$$

$$\vec{E}_1=\vec{E}_{inc}+\vec{E}_{sc} \qquad (2)$$

where μ_{r1} is the relative permeability and ϵ_{r1} the relative permittivity of air. k_0 is the wave numer of free space. $\vec{E}_1$ is the total electric field in the air domain. $\vec{E}_{inc}$ is the electric field of incident wave and $\vec{E}_{sc}$ is the scattered electric field. In the sea ice domain, the total field formulation is used. The wave equation is

$$\nabla\times\mu_{r2}^{-1}\left(\nabla\times\vec{E}_2\right)-k_0^2\epsilon_{r2}\vec{E}_2=0 \qquad (3)$$

where μ_{r2} is the relative permeability and ϵ_{r2} the relative permittivity of sea ice. $\vec{E}_2$ is the total electric field in the sea ice domain. The two different physics are coupled through the boundary condition on the interface between air and sea ice. The boundary condition is the continuity of tangential component of the electric field

$$\vec{n}\times\vec{E}_1=\vec{n}\times\vec{E}_2 \qquad (4)$$

where $\vec{n}$ is the outward normal of the interface.

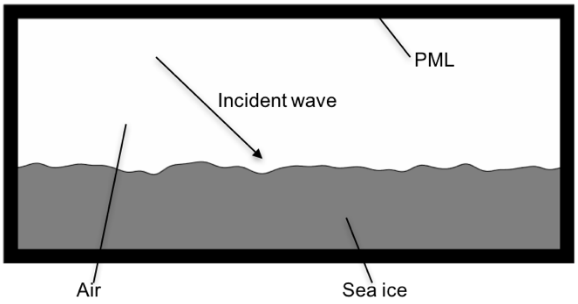

Figure 1. Geometry of the nonperiodic model

3.2 Implementation in COMSOL

To solve the problem described by equation (1-4), the RF module is utilized and two Electromagnetic Waves, Frequency Domain physics interfaces are selected. The first interface solves for the scattered field in the air domain, while the second accounts for the total field in the sea ice domain. The Electric Field node is created under the two interfaces and used for including the boundary condition (4). In the first physics interface, the incident wave needs to be defined for the scattered-field formulation. If the simulated surface is not infinite periodic, a plane incident wave can cause errors due to reflections from the surface edges. To solve the problem, a tapered incident wave defined in [5] is used. For the case of horizontal (transverse electric) polarization, the incident electric field is

$$\vec{E}_{inc}=E_b\vec{z} \qquad (5)$$

where

$$E_b=\exp\left\{-jk_0(x\sin\theta_i-y\cos\theta_i)\left[1+\frac{\left(\frac{2(x+y\tan\theta_i)^2}{g^2}-1\right)}{(k_0g\cos\theta_i)^2}\right]-\frac{(x+y\tan\theta_i)^2}{g^2}\right\}. \qquad (6)$$

θ_i is the incident angle of the incident wave, and g is the tapering parameter. We choose $g=L/5$ in this study, where L is the length of the rough surface. This choice is sufficient for ensuing that the tapered wave approximates the solution of the wave equation. For the case of the vertical (transverse magnetic) polarization, the incident electric field is

$$\vec{E}_{inc}=E_b\cos\theta_i\vec{x}+E_b\sin\theta_i\vec{y}. \qquad (7)$$

It should be noted that the coordinate system of the 2-dimensional model in COMSOL is x-y system, z is the direction for the out-of-plane. The incident electric field is implemented by setting the background electric field in COMSOL.

To truncate the computational domain, the Perfectly matched layer (PML) is used to surround the air and sea ice domain. The type of the PML is Cartesian and the coordinate stretching type is polynomial. The scaling factor and scaling curvature parameter of the PML are set as 1 by default. The thickness of the PML is set as one wavelength, which shows a good result. Further test should be done to check how to choose the smallest thickness for the model. Since we are not interested in the transmitted waves and these waves are absorbed by the PML, the bottom side of the PML can be put very close to the rough surface. Theoretically, there are very few waves reflected from the PML, especially when the incident angle approaches the grazing angle. To ensure that none of the waves influence the scattered wave, the bottom of the PML is put two wavelengths away from the rough surface. The top side of the PML is placed at least half a wavelength away from the rough surface to ensure no evanescent waves reach the PML.

For the meshing of the model, physics-controlled mesh is used to generate the mesh automatically. The largest size of the element is set as $\lambda/5$. The same meshing strategy is applied to the periodic model.

To calculate the radar cross section, the scattered field at near filed should be transformed to far field. The near-to-far field transformation can be implemented in COMSOL by adding a far-field calculation node. The air domain is selected as the far-field calculation domain and the scattered electric fields at the internal boundary of the PML are chosen for transformation. The calculation is implemented based on the Stratton-Chu formulation in COMSOL.

The bistatic RCS section is defined as

$$\sigma(\theta_s) = \lim_{r\to\infty} \frac{2\pi r\left|E_{sf}\right|^2}{gcos\theta_i\sqrt{\frac{\pi}{2}}\left[1-\frac{1+2tan^2\theta_i}{2k_0^2g^2sin^2\theta_i}\right]} \qquad (8)$$

where θ_s is the scattered angle, and E_{sf} is the electric field of the scattered wave at far field. It can be expressed in db scale through

$$\sigma_{db}(\theta_s) = 10log_{10}\sigma(\theta_s). \qquad (9)$$

The calculated RCS is random because the rough surface is generated based on the random theory. To remove the randomness, the Monte Carlo method is used to take the ensemble average of simulated field results. A number of different rough surfaces with the same roughness parameters are generated. The scattered fields are calculated for every realization. Results are averaged to get the final RCS. This is achieved by using the LiveLink™ for MATLAB®. Since the simulation is conducted for many surface instances independently, parallel computing can be used to accelerate the simulation.

4. Periodic model

In this study, the periodic model is built based on the total field formulation. Only one Electromagnetic Waves, Frequency Domain physics interface is used to account for the computational domain. The incident wave is excited by setting the port boundary conditon on the top of the model, as is shown in Figure 2. The type of the port is periodic. Floquet periodic boundary conditions are applied on the left side T_2 and right side T_3 to simulate a periodic rough surface. In this case, the plane wave launched by the port does not cause errors from the edge reflection because the surface is periodic. A PML layer is located on the bottom to absorb the transmitted waves.

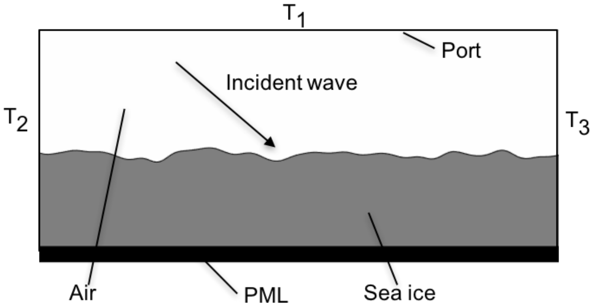

Figure 2. Geometry of the periodic model

According to the Floquet's theory, the scattered waves propagate along certain discrete angles (Bragg angles). To receive all the scattered waves in different directions, several diffraction order ports need to be added. The number of the ports m is decided by the surface period d, the incident angle θ_i and the wavelength λ. It can be calculated following

$$dsin\theta_s - dsin\theta_i = m\lambda$$

$$sin\theta_s = \frac{m\lambda}{d} + sin\theta_i$$

$$-1 \le \frac{m\lambda}{d} + sin\theta_i \le 1$$

$$-\frac{(1+sin\theta_i)d}{\lambda} \le m \le \frac{(1-sin\theta_i)d}{\lambda}$$

In COMSOL, one can create the appropite number of ports automatically. These diffraction ports can be put on the boundary T_1. The ports only receives the propagating waves and reflects the evanescent waves, so the boundary T_1 should be placed at least half a wavelength away from the rough surface.

The power of the incident wave is set as 1W. Based on the definition in [3], the bistatic RCS for the periodic rough surface is defined as

$$\sigma(\theta_s) = dk_0|S_{n1}|^2cos\theta_s \qquad (10)$$

where S_{n1} is the S-parameter from port n to port 1 and defined as

$$S_{n1} = \sqrt{\frac{Power\ delivered\ to\ port\ n}{Power\ incident\ on\ port\ 1}}. \qquad (11)$$

The Monte Carlo method is also used in the same way as in the nonperiodic model.

5. Simulation Results

The two models are tested for the C band microwave (wavelength $\lambda = 5.6$cm). A typical dataset for newly formed sea ice is chosen and the complex permittivity is calculated as 5.5-0.2i. The rms height of sea ice surface and corrlection length are set as 0.002m and 0.02m which are within the realistic range. By performing Monte Carlo simulations, a total of 150 sea ice surfaces are generated and the ensemble-averaged bistatic RCS is calculated for the incident angle 30°. In this study, only results of HH polarization are shown.

For the nonperiodic model, the surface length is set as 36 λ**.** Figure 3 shows the magnitude of the electric field simulated by the nonperiodic model. For the periodic model, the surface length (period) is set as 10 λ**.** Figure 4 shows the magnitude of the electric field simulated by the nonperiodic model. The scale and range of the two models are different because the power of the incident waves are not the same. It is clear that the tapered wave almost disappeared at the edges of the interface, while the plane wave is distributed everywhere.

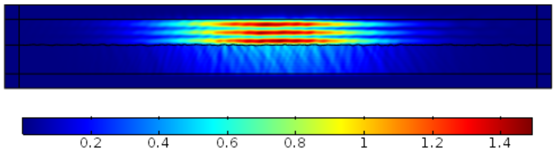

Figure 3. Magnitude of the electric field in the nonperiodic model

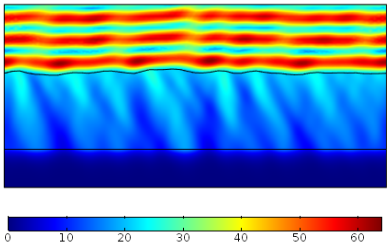

Figure 4. Magnitude of the electric field in the periodic model

The simulated bistatic RCS results are shown in Figure 5. The results are compared with the commonly used method SPM. Good agreements are achieved between all the three models. The only difference is at the specular direction. This is because the SPM model only considers the incoherent scattering. Compared with the unperiodic model, the periodic model can only simulate the scattered waves at certain Bragg angles (marked with green circle). In this case, the periodic model can not simulate the scatterd wave if the scattered angle is larger than 70°.

The advantage of the periodic model is that a shorter rough surface is sufficient for scattering simulation, which decreases the computational resource. This differs to the nonperiodic surface that should be large enough to include the incident wave with a large incident angle and account for the statistical property. To examine the effect of the surface length in the periodic model, two different surface sizes are used to simulate the bistatic RCS. As shown in figure 6, similar results are achieved for surface length at 10λ and 5λ. It means the width of the computational domain can be decreased to 5λ**.** The smaller size can cause a larger width and less power in the specular direction. There are also less diffraction orders for shorter surface, which means there are less Bragg angels. In this case, the surface with 5λ can not simulate the scattered wave if the scattered angle is larger than 50°, while the surface with 10 λ can account for the scattered angle up to 60°.

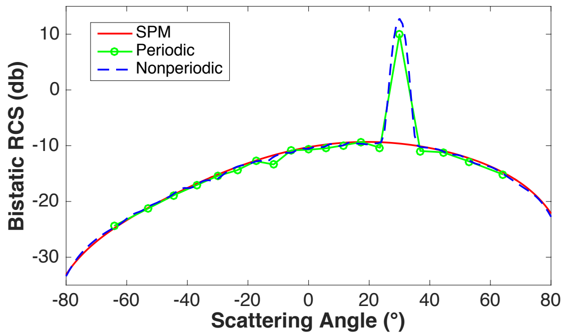

Figure 5. Bistatic RCS simulated by the nonperiodic model, the periodic model and SPM

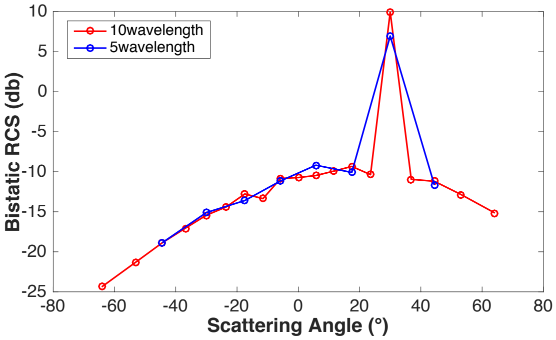

Figure 6. Bistatic RCS simulated by the periodic model at different surface sizes

The above tests are performed on a MacBook Pro with 16 GB memory and the time for calculating the scattering of one surface instance is less than 15s. Parallel computing is also applied on Monte Carlo simulation, which decreases the total computational time.

6. Conclusions

In this study, a nonperiodic model and a periodic model are built to simulate the microwave scattering from the sea ice surface by the use of COMSOL. The Monte Carlo method is included through LiveLink™ for MATLAB. Compared with the SPM, the simulated results of the two models have good agreements. The periodic model requires a much smaller computational domain, which is important for the development of a 3-D scattering model in the future.

References

1. H. Lawrence, F. Demontoux, J. P. Wigneron, P. Paillou, T. D. Wu, and Y. H. Kerr. "Evaluation of a numerical modeling approach based on the finite-element method for calculating the rough surface scattering and emission of a soil layer." IEEE Geoscience and Remote Sensing Letters 8.5 (2011): 953-957.
2. U. K. Khankhoje, J. J. van Zyl, and T. A. Cwik, "Computation of Radar Scattering From Heterogeneous Rough Soil Using the Finite-Element Method," Ieee Transactions on Geoscience and Remote Sensing, vol. 51, pp. 3461-3469, Jun 2013.
3. L. Tsang, J.A. Kong, K.H. D, and C.O. Ao, Scattering of electromagnetic waves: Numerical Simulations, John Wiley & Sons, 2001.
4. Polder and J. H. Van Santeen, "The effective permeability of mixtures of solids," Physica , vol.12, pp. 257-271, 1946.figs
5. E.I. Thorsos, "The validity of the Kirchhoff approximation for rough surface scattering using a Gaussian roughness spectrum," The Journal of the Acoustical Society of America, vol. 83, pp. 78-92, 1998.