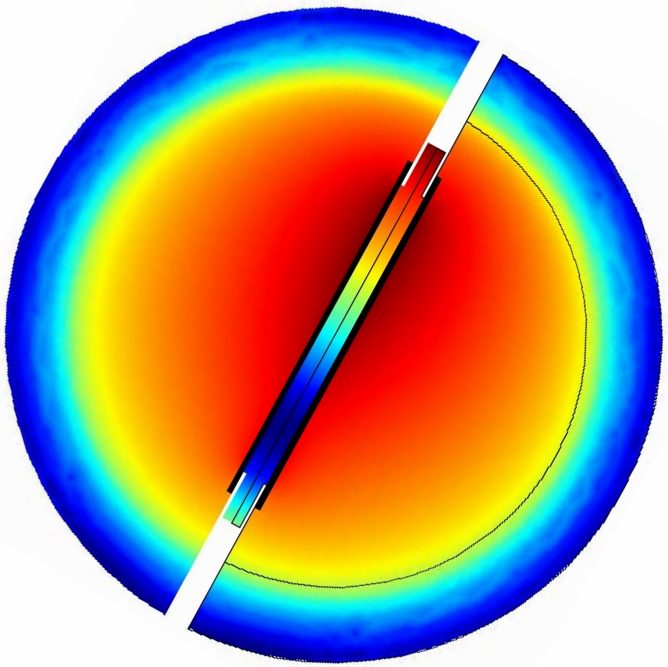
Sound attenuation by hearing aid earmold tubing



Comsol conference 2008

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Widex A/S, Denmark

Contents

- Motivation
- Modeling
  - Model setup
  - Theory … hopefully not too many equations
  - Boundary conditions
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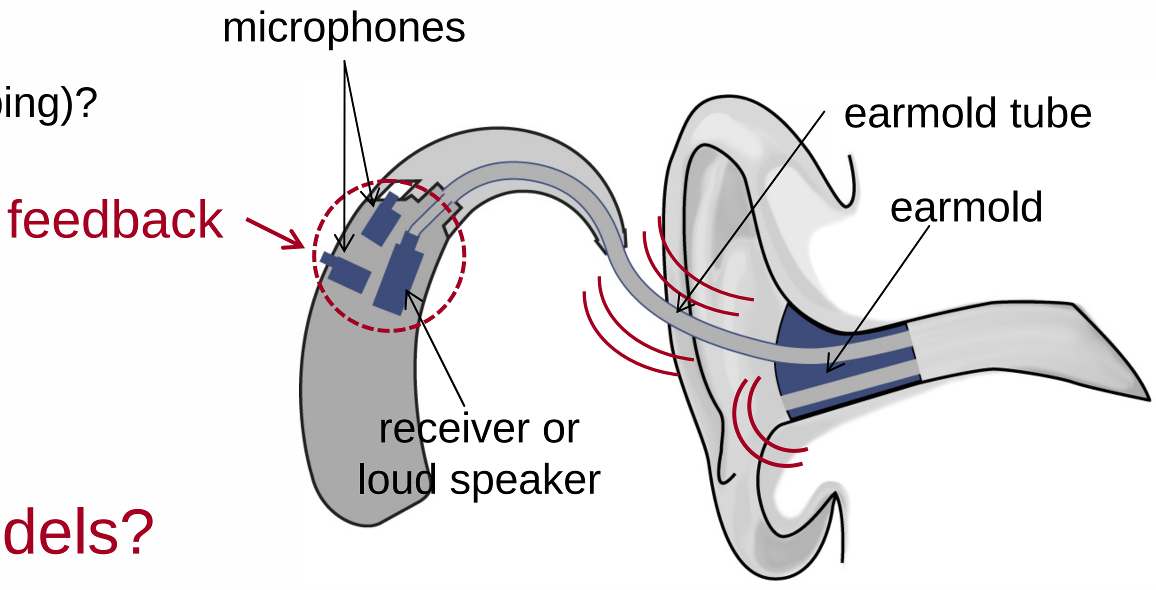
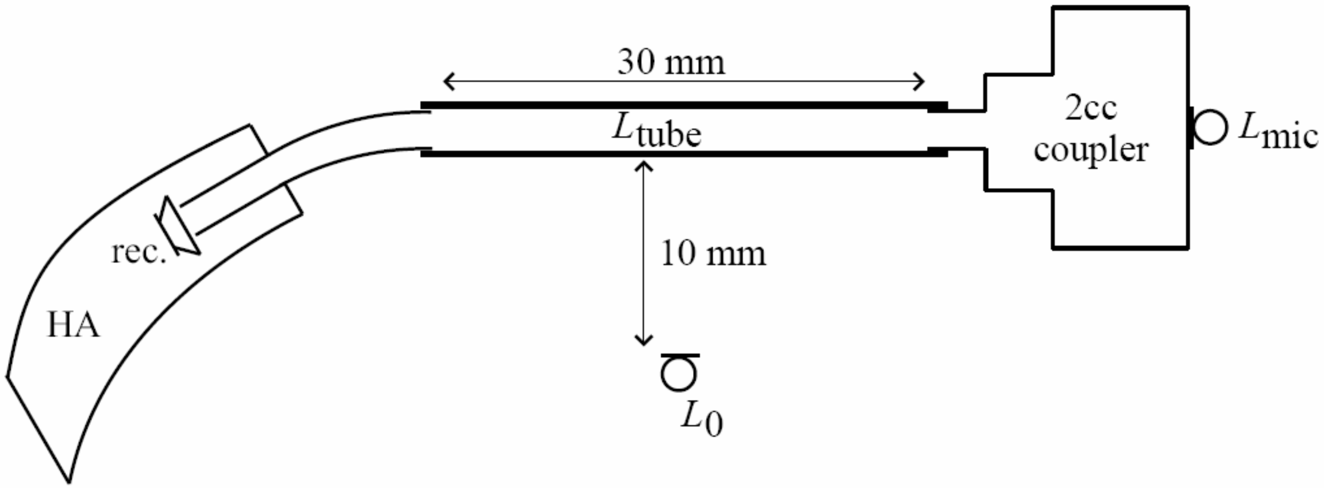
Motivation: why "sound attenuation"

- Introductory study in preparation for modeling a full hearing aid device
- Feedback in hearing aids:
  - Mechanical stability
  - Acoustic feedback:
    - Leaks and/or vent
    - Sound radiation (tubing)?
- Thicker tubes?
- Other materials?
- Include in future models?



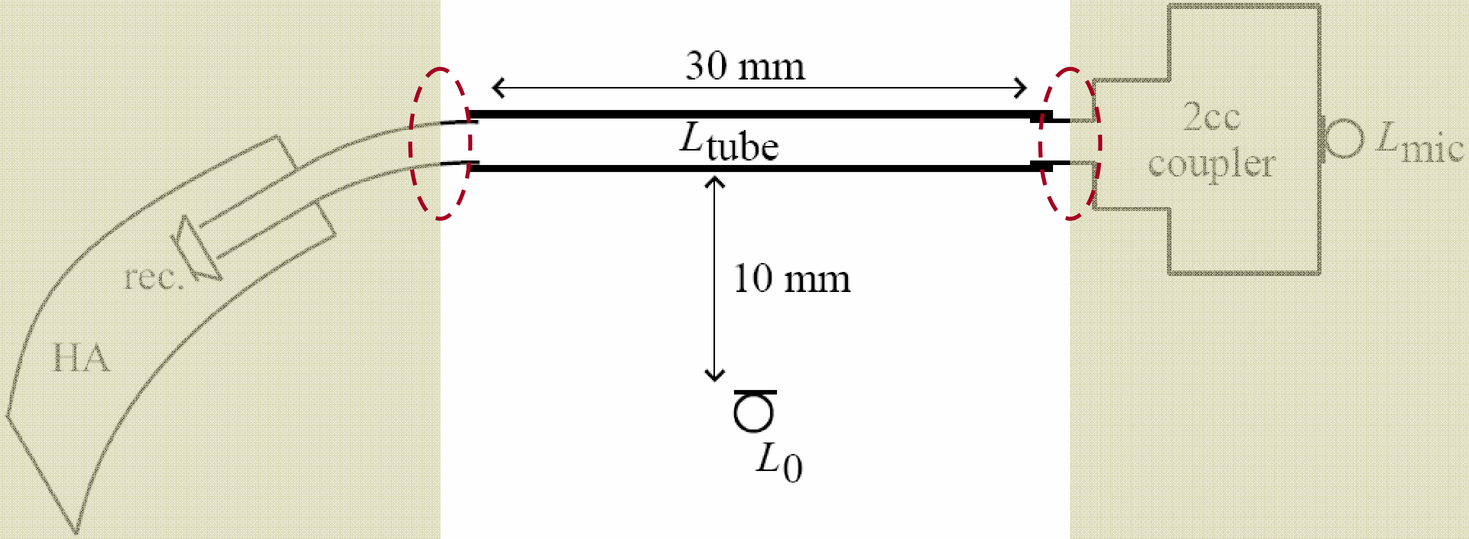
Virtual measurement set-up



- Real attenuation = $L_{tube} - L_0$
- Measured attenuation = $L_{mic} - L_0$

L. Flack, R. White, J. Tweed, D.W. Gregory, and M.Y. Qureshi
Brit. J. of Aud., **29**, 237 (1995)

Model setup



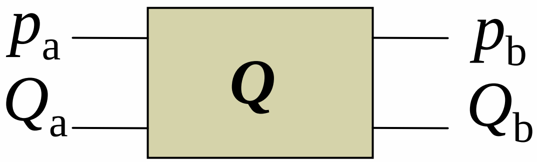
In: Electro-acoustic equivalent

FEM domain:
Fluid (air)
Solid (tube)
PML

Out: Electro-acoustic equivalent

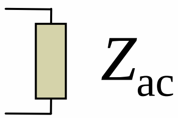
Theory: electroacoustic model

- Electric analogue to an acoustic system



$$\begin{bmatrix} p_a \\ Q_a \end{bmatrix} = \boldsymbol{Q} \begin{bmatrix} p_b \\ Q_b \end{bmatrix} \text{ where } \boldsymbol{Q} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

- An acoustic system may also be terminated by an acoustic impedance Z_{ac}



$$Z_{ac} = Z_{ac}(f) = \frac{\bar{p}}{Q}$$

- For this model to hold we assume plane waves!

Theory: thermoviscous acoustics

- Assuming harmonic variations a small parameter expansion of the Navier-Stokes, continuity, and the energy equation yields:

$$i\omega\rho_0\boldsymbol{u} = -\nabla p + (\lambda+\mu)\nabla(\nabla\cdot\boldsymbol{u}) + \mu\nabla^2\boldsymbol{u}$$

$$i\omega\left(\frac{p}{p_0} - \frac{T}{T_0}\right) = -\nabla\cdot\boldsymbol{u}$$

$$i\omega\rho_0 C_v T = \kappa\nabla^2 T - p_0\nabla\cdot\boldsymbol{u}.$$

$$p_0 = R_0\rho_0 T_0$$

Theory: elastic waves in solids

- Assuming small deformations in a solid
  - Momentum:

$$-\omega^2 \rho_0 U_i = \nabla_j \sigma_{ij}$$

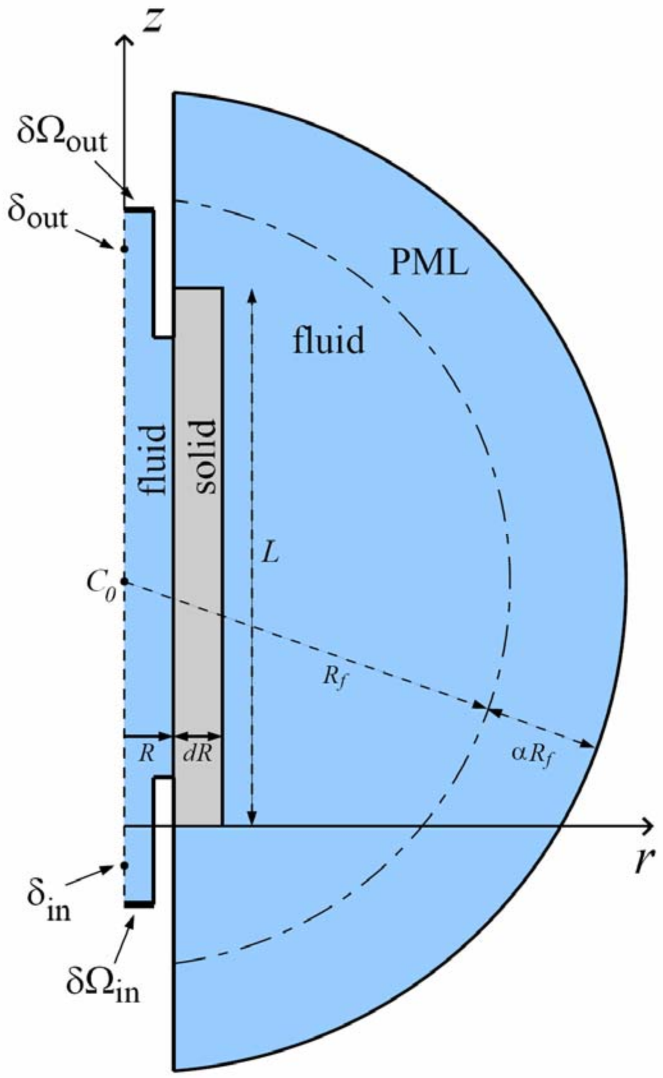
  - Stress (elastic) and strain tensor (linearized):

$$\sigma_{ij} = 2\mu_s \varepsilon_{ij} + \lambda_s \delta_{ij} \varepsilon_{kk} \qquad \varepsilon_{ij} = \frac{1}{2}(\nabla_i U_j + \nabla_j U_i)$$

  - Modeling losses Young's modulus is represented as

$$\tilde{E} = E(1 + \eta i)$$

FEM domain (axisymmetric)



measured properties of solid

σ = 0.45 (Poisson ratio)

E = 4.1 10^7 Pa (Young's modulus)

η = 0.019 (loss factor)

ρ = 1220 kg/m^3 (density)

$\hat{E}$ = E(1 + ηi) (complex Young's modulus)

other parameters

Frequency: f [Hz]

Temperature: T [K]

Atmospheric pressure: p = 10^5 Pa

Density: ρ [kg/m^3]

Speed of sound: c [m/s]

Dynamic viscosity: μ [Pa·s]

Heat conductivity: κ [W/(m · K)]

Specific heat (@ const. p): Cp [J/(kg·K)]

Ratio of specific heats: γ = Cp/Cv

Weak formulation for FEM and PML

- Governing (fluid domain)

$$\int_{\Omega} \left[\left(i\omega\left(\frac{p}{p_0} - \frac{T}{T_0}\right) + \nabla_i u_i\right)\tilde{p} + i\omega\rho_0 u_j \tilde{u}_j + \sigma_{ij}\nabla_j \tilde{u}_i + i\omega(\rho_0 C_p T - p)\tilde{T} + \kappa\nabla_i T \nabla_i \tilde{T} \right] dA = 0$$

- Perfectly matched layer (PML, open boundary)

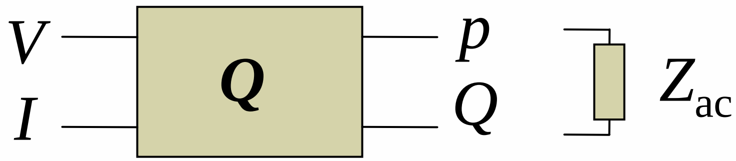
$p = e^{i\omega - kx} \quad x \rightarrow a + bi \quad \tilde{p} = e^{i\omega - ka - bki}$.

attenuated by the amount e^{-bki}.

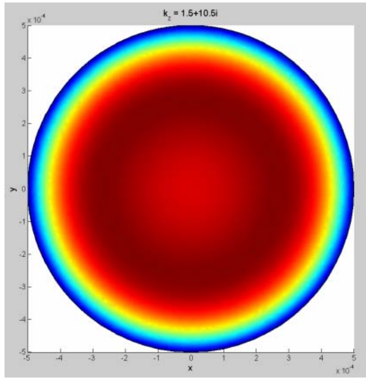
$$\int_{\Omega} f(x_i, \frac{\partial}{\partial x_i}) dV \rightarrow \int_{\tilde{\Omega}} f(Z_{x_i}, \frac{\partial}{\partial Z_{x_i}}) d\tilde{V} = \int_{\Omega} f(Z_{x_i}(x_i), J_{ji}^{-1}\frac{\partial}{\partial x_j}) |J| dV.$$

AIBC: Inlet boundary condition

- Electroacoustic relation for BC
- Requires plane wave at inlet $\partial\Omega_e$



- Solve for the non-evanescent eigen-solution $\boldsymbol{u}_e$ on inlet $\partial\Omega_e$

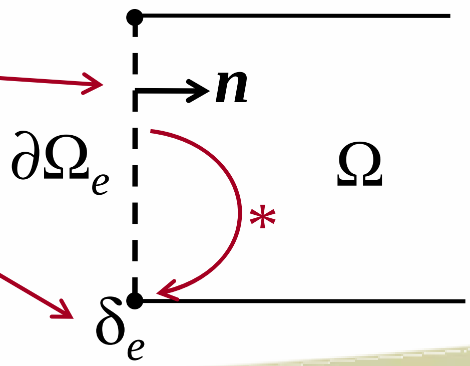


- Apply BC as weak constraint and scale $\boldsymbol{u}$ with Lagrange multiplier

$$\boldsymbol{u} - \lambda_1 \boldsymbol{u}_e = 0 \quad \text{on} \quad \partial\Omega_e$$

$$\int_{\delta_e} (V - A_{in}P_{in}/\alpha_{in} - B_{in}Q_{in})\tilde{\lambda}_1 \, dP = 0$$

$$[\alpha_{in}, P_{in}, Q_{in}] = \int_{\partial\Omega_e} [1, p, \boldsymbol{u}\cdot\boldsymbol{n}] \, dA^{*}$$



Other BCs

- Sound hard wall (isothermal):

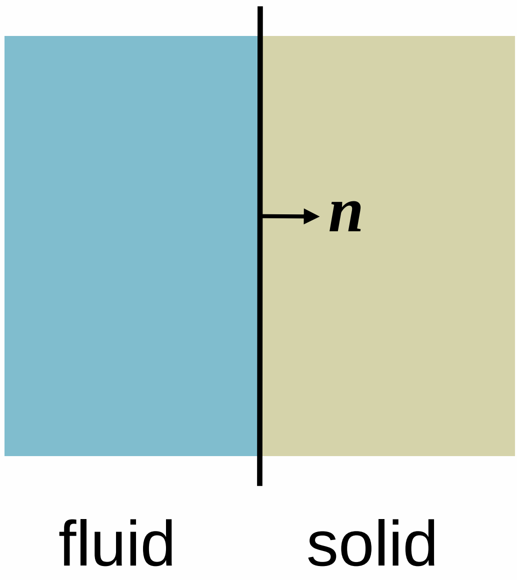
$$\boldsymbol{u} = \boldsymbol{0} \quad \text{and} \quad T = 0$$

- Solid fluid coupling (continuity of normal stress and displacement):

$$\boldsymbol{n} : \boldsymbol{\sigma} = \boldsymbol{n} : \boldsymbol{S}$$

$$\boldsymbol{u} = \frac{\partial \boldsymbol{U}}{\partial t} = i\omega \boldsymbol{U}$$

$$T = 0$$

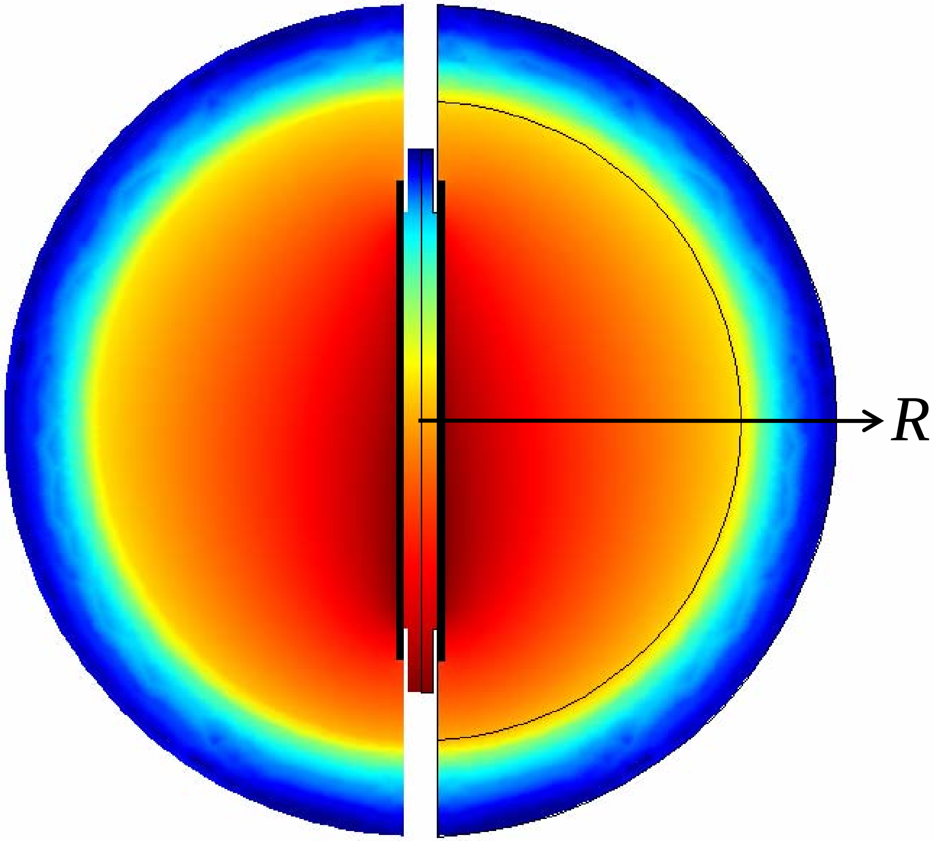


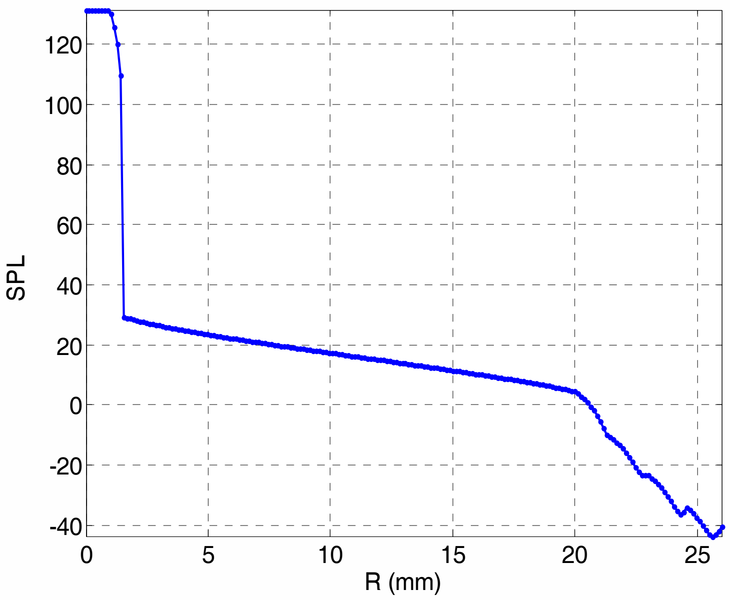
- Outlet BC as inlet BC (no source):

$$p = Q\, Z_{ac}$$

Results: sound pressure level

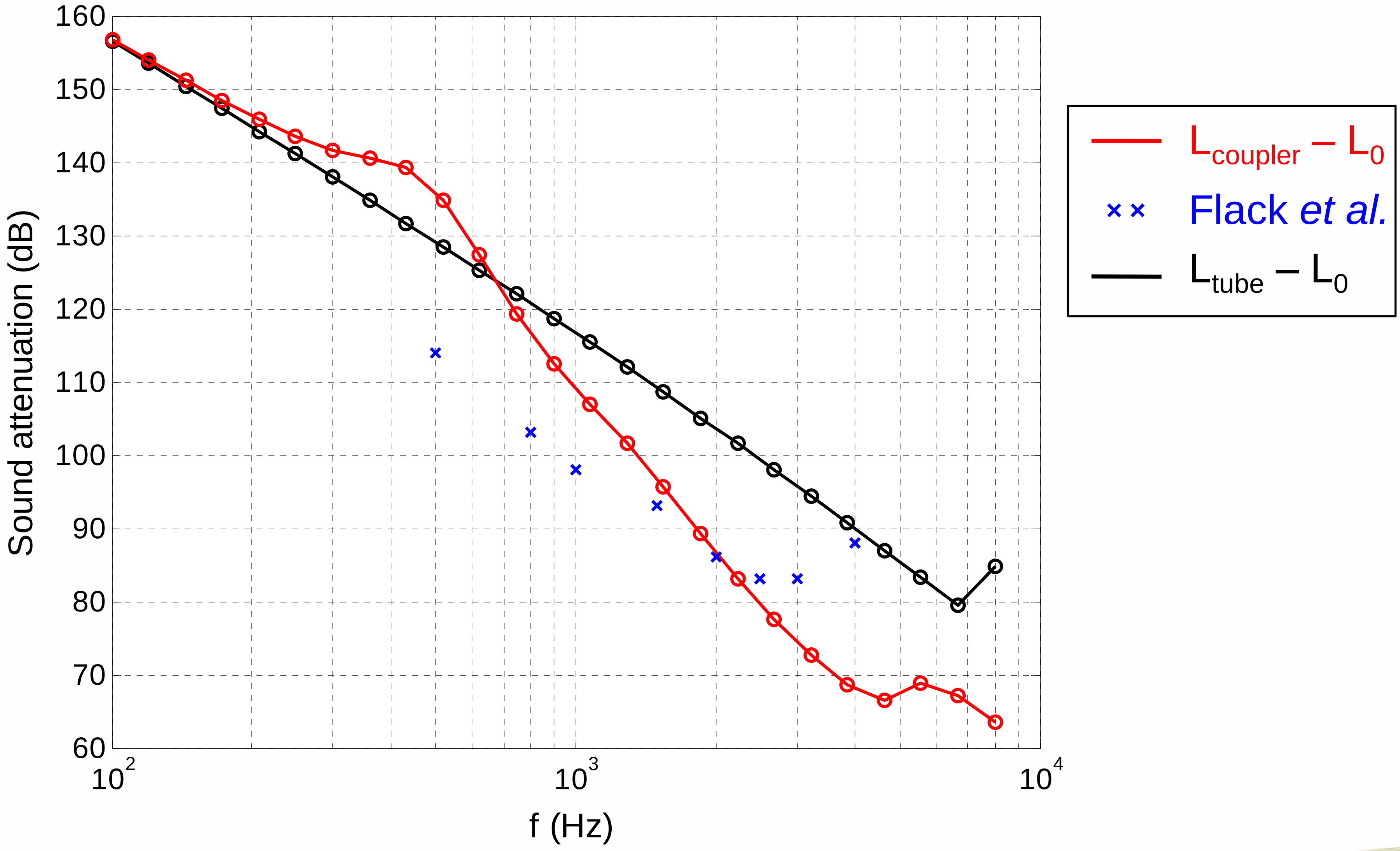
f = 1000 Hz





$L = 20 \log(p/p_{ref})$ where $p_{ref} = 20\ \mu\text{Pa}$

Results: sound attenuation



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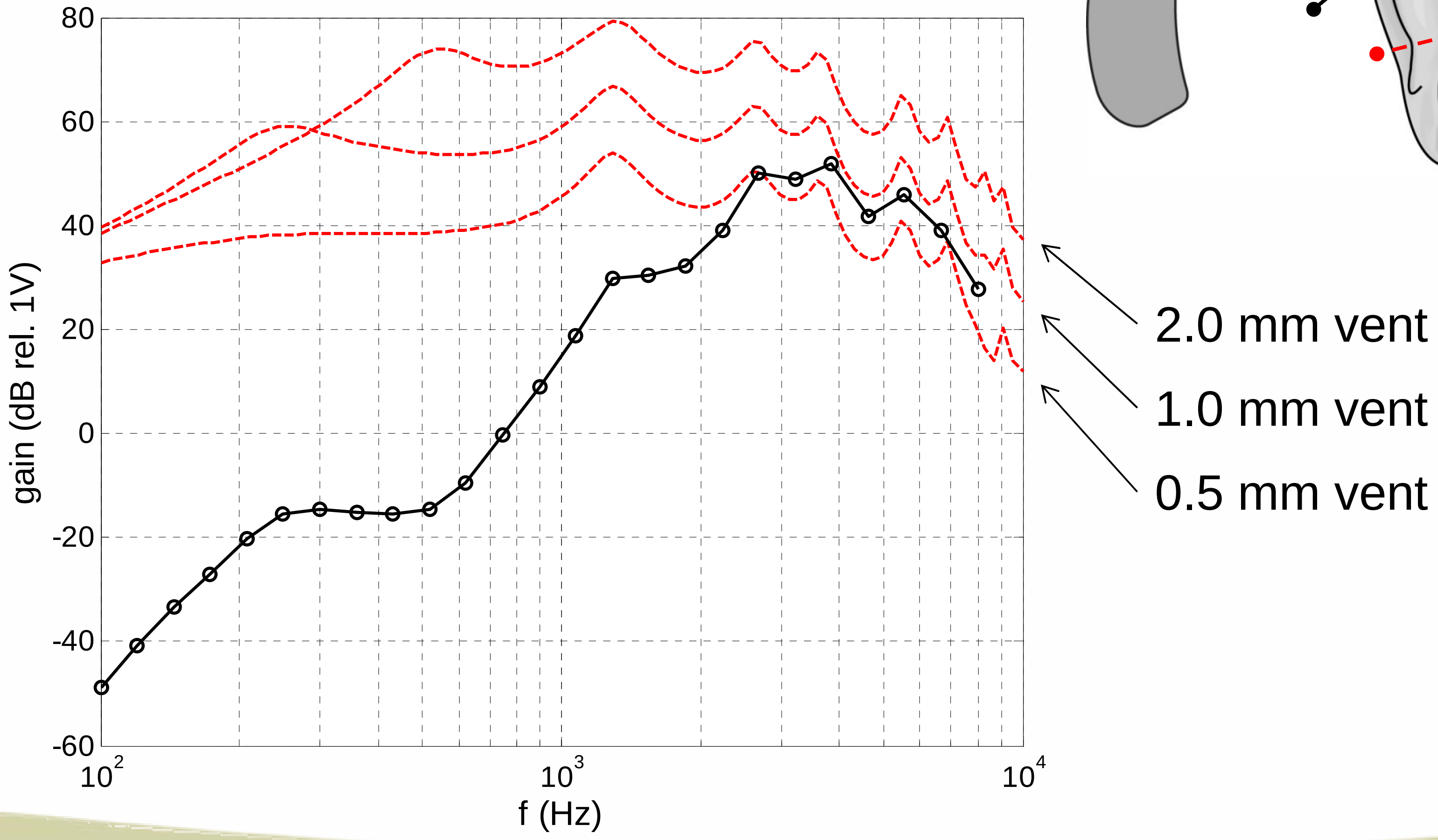
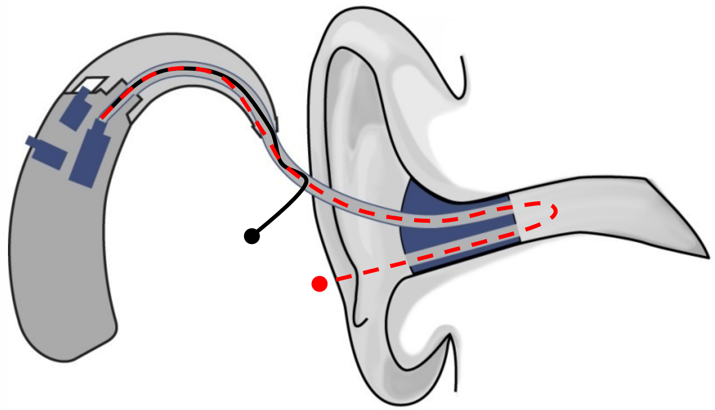
Results: acoustic feedback





WIDEX high definition hearing

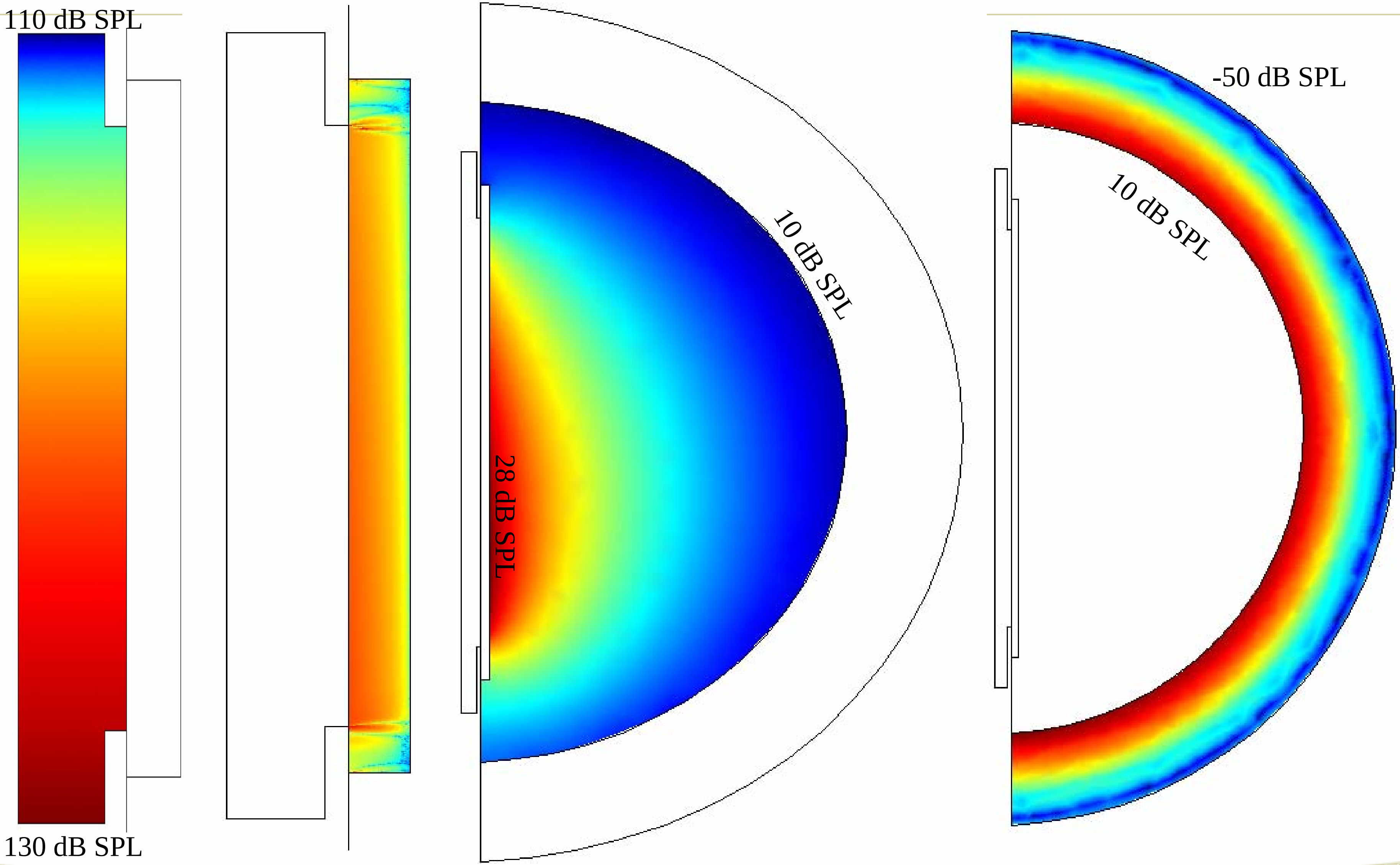
Conclusions

- 2D model to analyze sound radiation
- PML for thermoviscous acoustic system
- Coupling between electroacoustic model and FEM with AIBC

- Order of magnitude OK (Flack *et al.*)
- Attenuation is high for standard earmold tubing (> 80 dB)
- Feedback @ high frequencies? More detailed study needed.

- Some numerical effects/instabilities in the system – comments are welcome after the session!

Results: pressure / stress level



$L = 20 \log(p/p_{ref})$ where $p_{ref} = 20\ \mu\text{Pa}$ $f = 1000$ Hz