Transformation Optics Simulation Method for Stimulated Brillouin Scattering

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R. Zecca *et al.*, *Phys. Rev. A* (2016), in review

Stimulated Brillouin scattering

- Nonlinear coupling of light and elastic waves (optical forces, scattering)
- Applications: optical lasers, amplifiers, strain sensors, slow light
- Potential for high gains and SNR in (nano-)structured materials/devices
- Numerical modeling is key to device design

Finite-element SBS modeling

Stokes process: $\omega_{\mathrm{p}} = \omega_{\mathrm{s}} + \Omega$

MECHANICS

Displacement vectorial field

$$\tilde{\mathbf{u}} = \mathbb{R}\mathrm{e}\left(\mathbf{u}\, e^{i\Omega t}\right)$$

Mass density variation scalar field

$$\Delta\tilde{\rho} = \mathbb{R}\mathrm{e}\left(\Delta\rho\, e^{i\Omega t}\right)$$

$$\Delta\rho \propto \nabla \mathbf{u}$$

OPTICS

Bi-chromatic electromagnetic field

$$\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_{\mathrm{p}} + \tilde{\mathbf{E}}_{\mathrm{s}}$$

$$\tilde{\mathbf{H}} = \tilde{\mathbf{H}}_{\mathrm{p}} + \tilde{\mathbf{H}}_{\mathrm{s}}$$

$$\tilde{\mathbf{E}}_n = \mathbb{R}\mathrm{e}\left(\mathbf{E}_n e^{i\omega_n t}\right)$$

$$\tilde{\mathbf{H}}_n = \mathbb{R}\mathrm{e}\left(\mathbf{H}_n e^{i\omega_n t}\right)$$

$$n = \mathrm{p}, \mathrm{s}$$

Finite-element SBS modeling

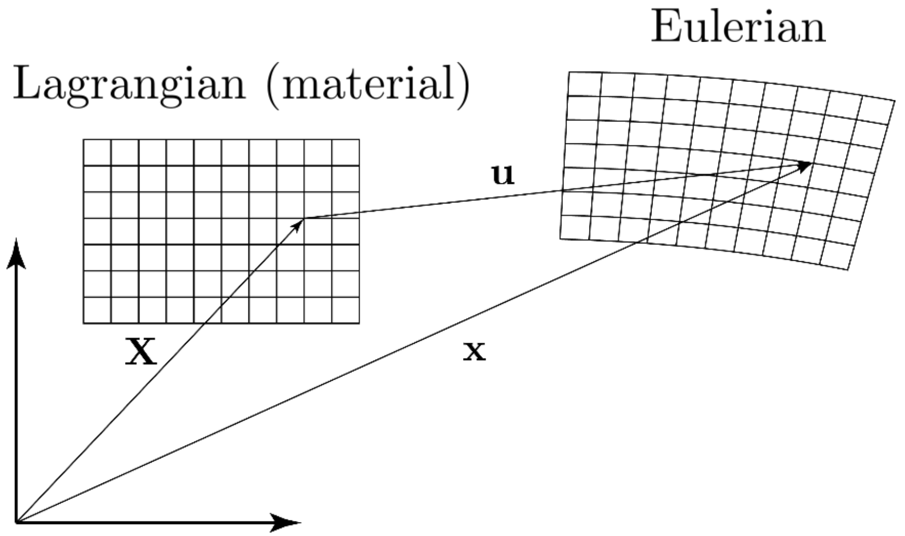
MECHANICS

mass x acceleration		internal force (stress)		optical forces (external)
$-\rho_0\,\Omega^2\mathbf{u}$	$=$	$\nabla_{\mathbf{X}}\cdot\left(\bar{\bar{F}}\,\bar{\bar{S}}\right)$	$+$	$\mathbf{f}\left(\nabla_{\mathbf{X}}\left(\mathbf{E}_{\mathrm{p}}\cdot\mathbf{E}_{\mathrm{s}}^{*}\right)\right)$

OPTICS

$$\nabla^2\mathbf{E}_{\mathrm{p}} + k_{\mathrm{p}}^2\,\mathbf{E}_{\mathrm{p}} = C_1\Delta\epsilon\left(\mathbf{u}\right)\mathbf{E}_{\mathrm{s}}$$

$$\nabla^2\mathbf{E}_{\mathrm{s}} + k_{\mathrm{s}}^2\,\mathbf{E}_{\mathrm{s}} = C_2\Delta\epsilon^{*}\left(\mathbf{u}\right)\mathbf{E}_{\mathrm{p}}$$

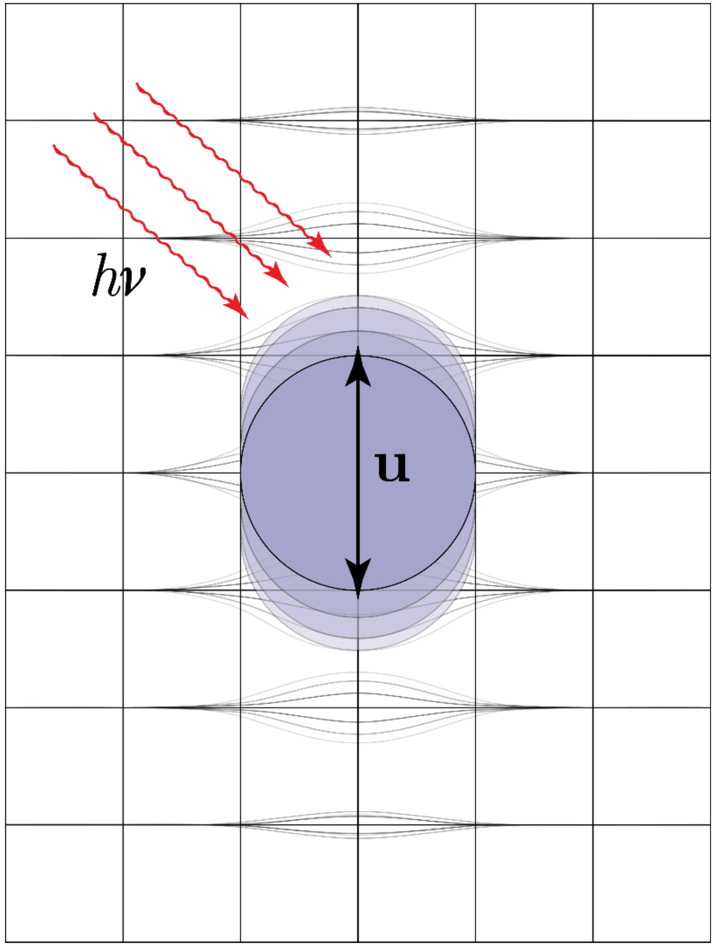


$$\mathrm{d}\mathbf{x} = \bar{\bar{F}}\mathrm{d}\mathbf{X}$$

$$\bar{\bar{F}} = \bar{\bar{F}}\left(\mathbf{u},\Delta\rho\right)$$

Finite-element SBS modeling

- Frequency-domain EM solver cannot take moving geometry into account
- Significant problem in the case of SBS
- Time-domain too onerous: $\frac{2\pi}{\Omega} \gg \frac{2\pi}{\omega}$ (10 ps vs. fs)
- Different strategy needed



Transformation optics: designing an invisibility cloak

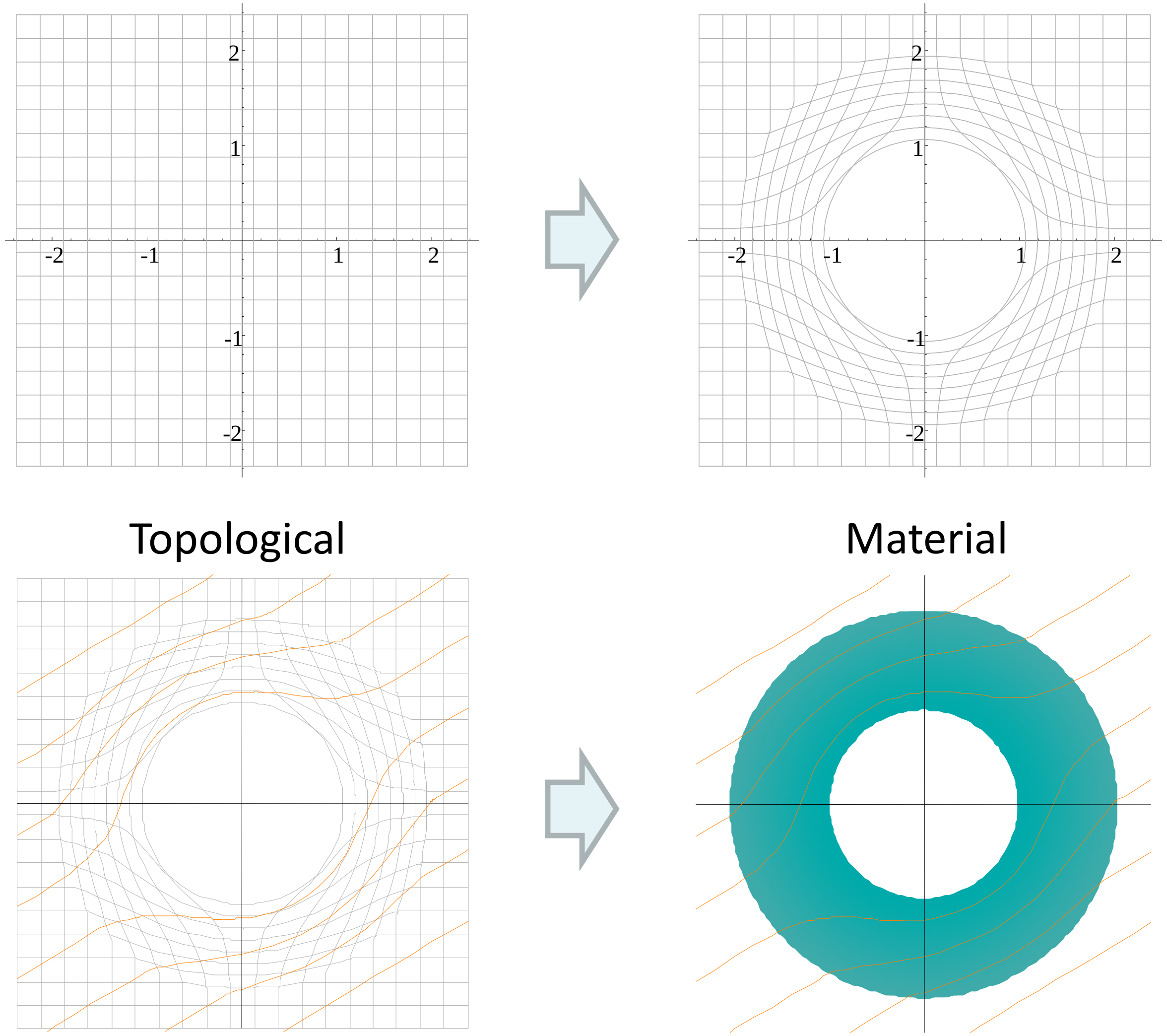
Given a coordinate transform

$$\mathbf{x} \rightarrow \mathbf{x}' \qquad \bar{\bar{\Lambda}} = \frac{\partial \mathbf{x}'}{\partial \mathbf{x}}$$

Using form-invariance of Maxwell's equations

$$\bar{\bar{\epsilon}}' = \frac{\bar{\bar{\Lambda}}\bar{\bar{\epsilon}}\bar{\bar{\Lambda}}^T}{\det \bar{\bar{\Lambda}}}$$

$$\bar{\bar{\mu}}' = \frac{\bar{\bar{\Lambda}}\bar{\bar{\mu}}\bar{\bar{\Lambda}}^T}{\det \bar{\bar{\Lambda}}}$$



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Time-dependent TO metric

- Represent movement by equivalent, time-dependent material properties
- Apply time-dependent metric, using deformation gradient $\bar{\bar{F}}$ as Jacobian

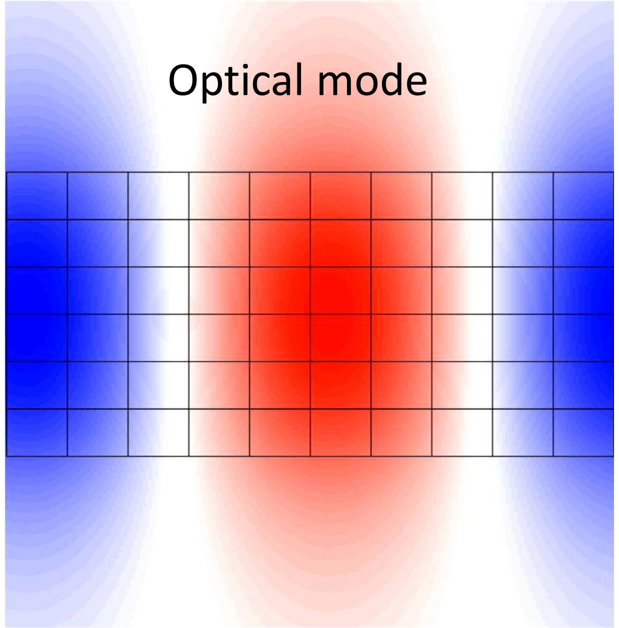
$$\bar{\bar{\epsilon}}' = \bar{\bar{g}}\,\epsilon$$
$$\bar{\bar{\mu}}' = \bar{\bar{g}}\,\mu$$

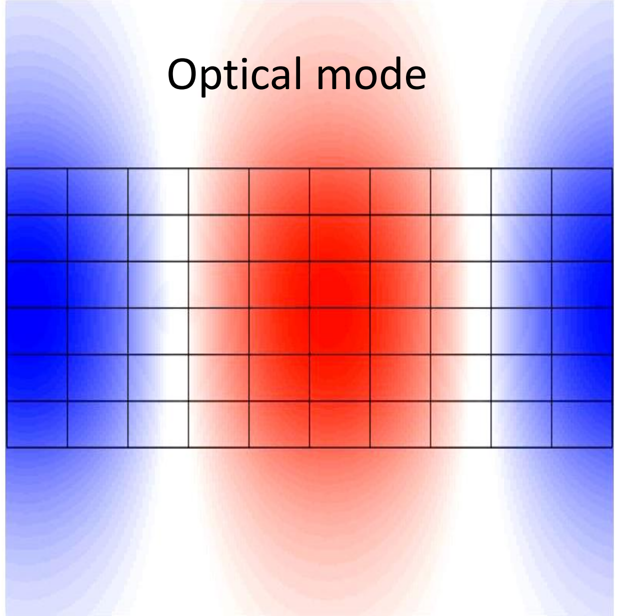
$$\bar{\bar{g}} = \frac{\bar{\bar{F}}\,\bar{\bar{F}}^T}{\det \bar{\bar{F}}}$$

$$\bar{\bar{g}} = \sum_{n=-3}^{3} \bar{\bar{g}}_n e^{in\Omega t}$$

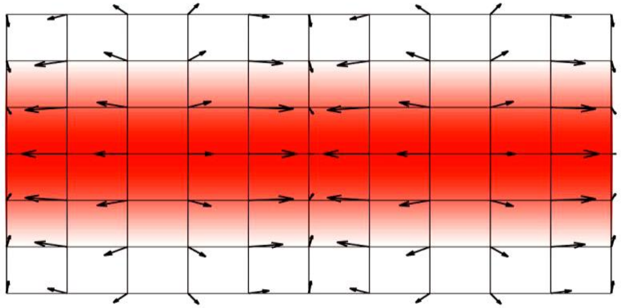
$$\bar{\bar{g}}_n = \bar{\bar{g}}^*_{-n}$$

$$\bar{\bar{g}} \rightarrow \bar{\bar{I}} \quad \text{for} \quad \mathbf{u} \rightarrow \mathbf{0}$$

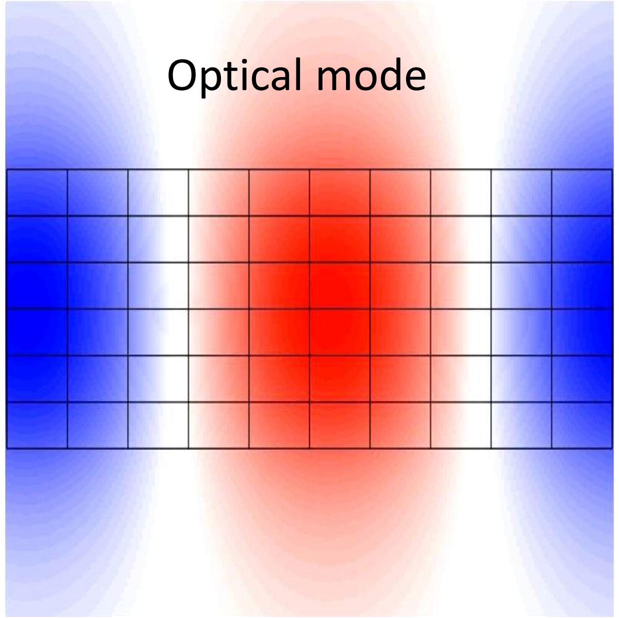

Optical mode



Optical forces and potential



Optical mode



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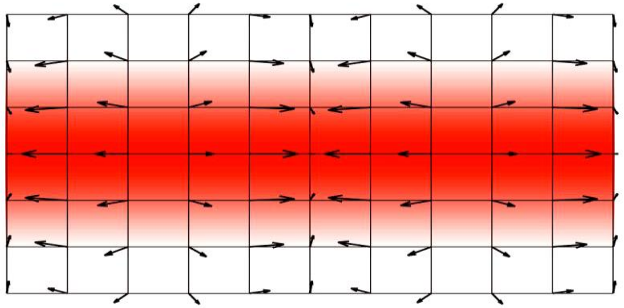
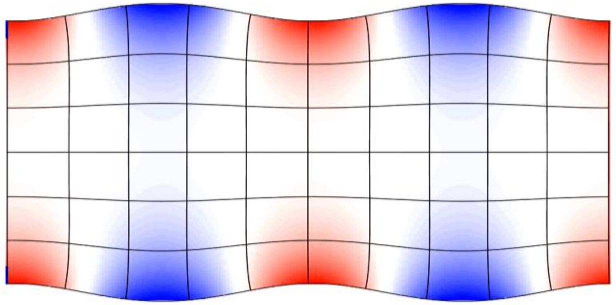
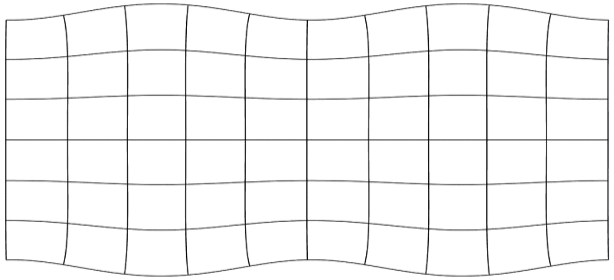


Mass density/permittivity variation



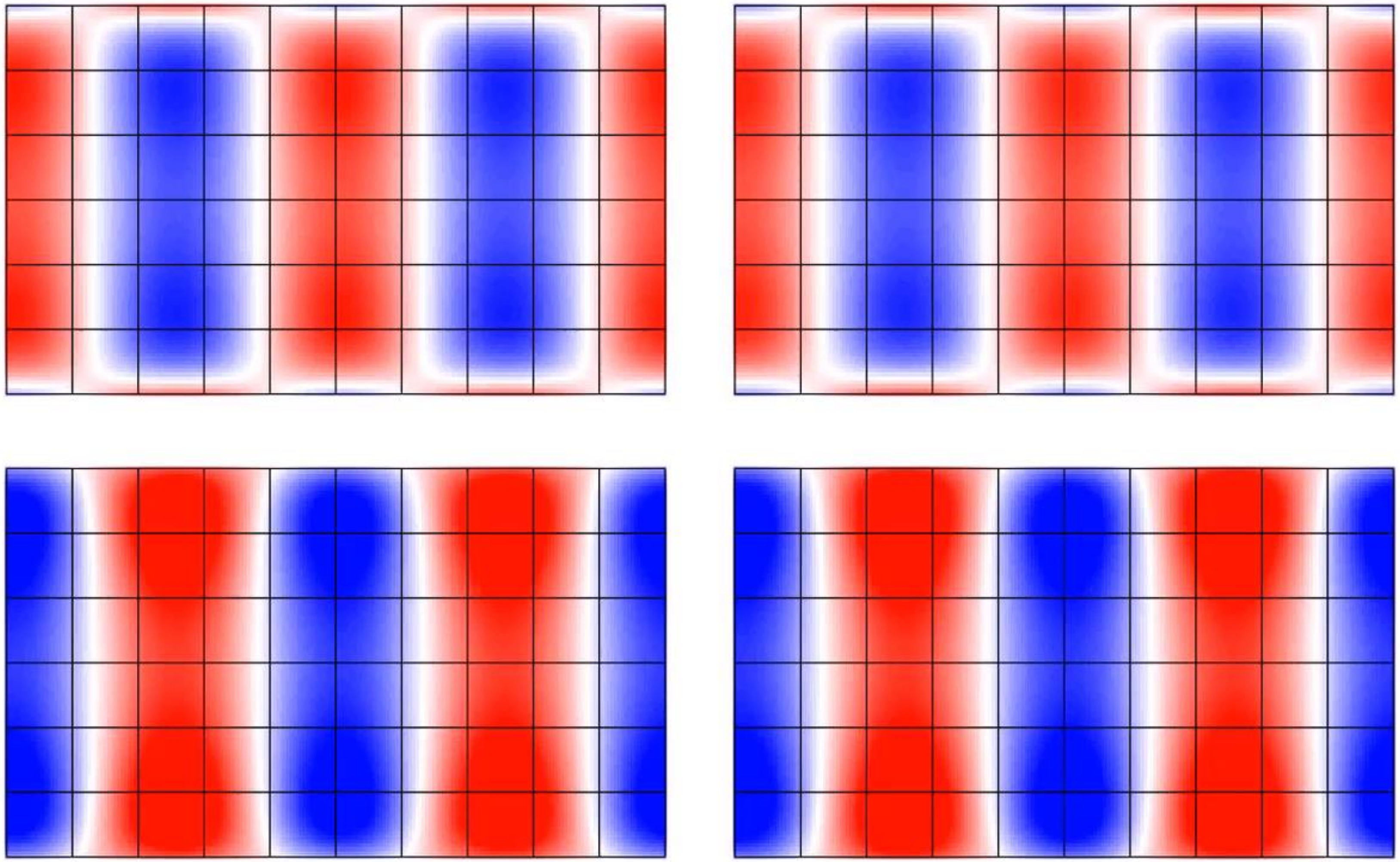
Permeability variation



$$\frac{\epsilon_r + \Delta\epsilon}{\epsilon_r} \qquad \frac{\mu_r + \Delta\mu}{\mu_r}$$

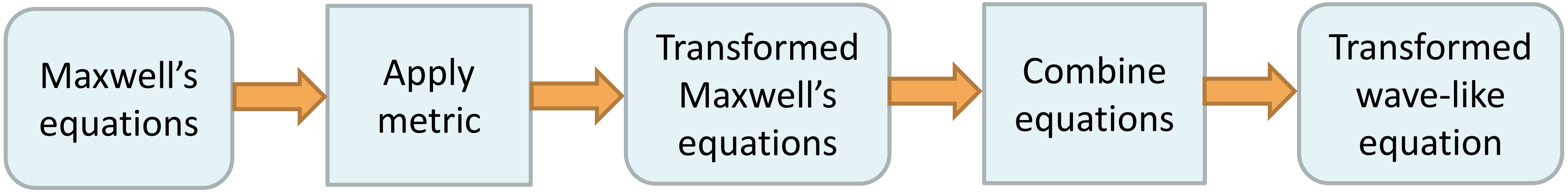
Longitudinal components

Transverse components



Transformed Maxwell’s equations

- COMSOL master equation $\nabla \times \left(\bar{\bar{\mu}}_r^{-1} \nabla \times \mathbf{E}\right) - k_0^2 \left(\bar{\bar{\epsilon}}_r - \frac{i\bar{\bar{\sigma}}_{\mathrm{e}}}{\omega\epsilon_0}\right)\mathbf{E} = \mathbf{0}$
- Apply TO metric to Maxwell’s equations and obtain similar expression



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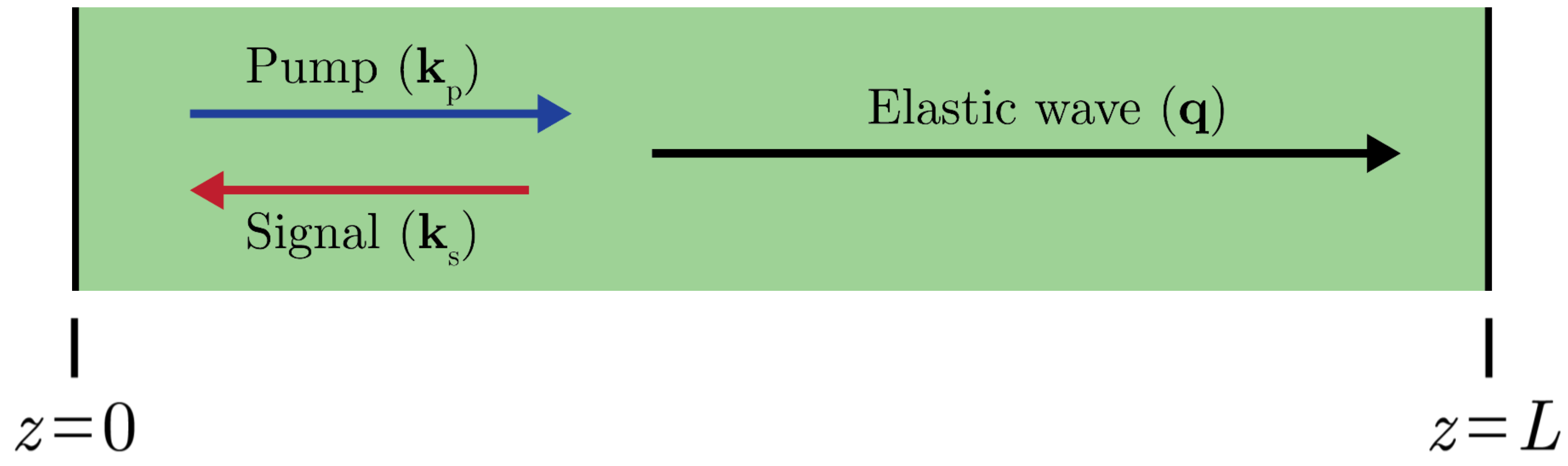
- Apply TO metric to Maxwell's equations and obtain similar expression

$$\nabla \times \bar{\bar{A}}^{-1} \nabla \times \mathbf{E}_{\mathrm{p}} - \omega_{\mathrm{p}}^2 \bar{\bar{C}} \mathbf{E}_{\mathrm{p}} = \omega_{\mathrm{p}}^2 \left[a\bar{\bar{K}} \mathbf{E}_{\mathrm{p}} + \left(\bar{\bar{D}} + a\bar{\bar{L}}\right) \mathbf{E}_{\mathrm{s}} \right] - i\omega_{\mathrm{p}} \nabla \times \left(\bar{\bar{A}}^{-1} \bar{\bar{B}}\, \mathbf{H}_{\mathrm{s}}\right)$$

$$\nabla \times \bar{\bar{A}}^{-1} \nabla \times \mathbf{E}_{\mathrm{s}} - \omega_{\mathrm{s}}^2 \bar{\bar{C}} \mathbf{E}_{\mathrm{s}} = \omega_{\mathrm{s}}^2 \left[\left(\bar{\bar{D}}^* + a\bar{\bar{L}}^*\right) \mathbf{E}_{\mathrm{p}} + a\bar{\bar{K}} \mathbf{E}_{\mathrm{s}} \right] - i\omega_{\mathrm{s}} \nabla \times \left(\bar{\bar{A}}^{-1} \bar{\bar{B}}^* \mathbf{H}_{\mathrm{p}}\right)$$

$\bar{\bar{A}} = \bar{\bar{g}}_0 \mu$ $\bar{\bar{B}} = \bar{\bar{g}}_1 \mu$ $\bar{\bar{C}} = \bar{\bar{g}}_0 \epsilon$ $\bar{\bar{D}} = \bar{\bar{g}}_1 \epsilon$ $\bar{\bar{K}} = \epsilon_0 \left(\bar{\bar{g}}_1 \Delta\epsilon^* + \bar{\bar{g}}_1^* \Delta\epsilon\right)/2$ $\bar{\bar{L}} = \epsilon_0 \left(\bar{\bar{g}}_0 \Delta\epsilon + \bar{\bar{g}}_2 \Delta\epsilon^*\right)/2$

Test: 1D bulk amplifier



Theory

Undepleted pump approximation

$$I_s(z) = I_s(L)\, e^{gI_p(L-z)}$$

R. W. Boyd, *Nonlinear Optics.*

TO method

Modified wave-like equations

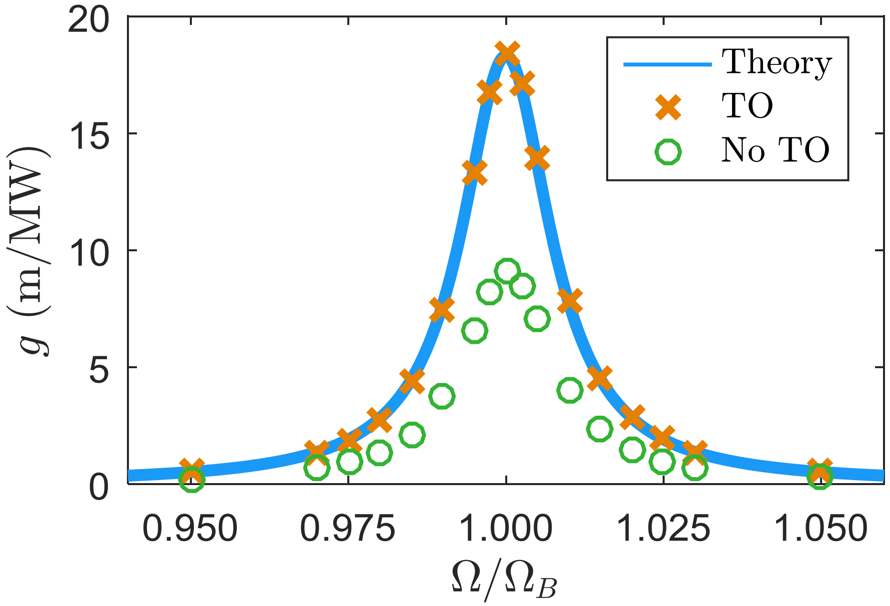
$$\bar{\bar{g}} = \sum_{n=-3}^{3} \bar{\bar{g}}_n e^{in\Omega t}$$

Simple coupling

Standard wave-like equations

$$\bar{\bar{g}} = \bar{\bar{I}}$$

Test: 1D bulk amplifier



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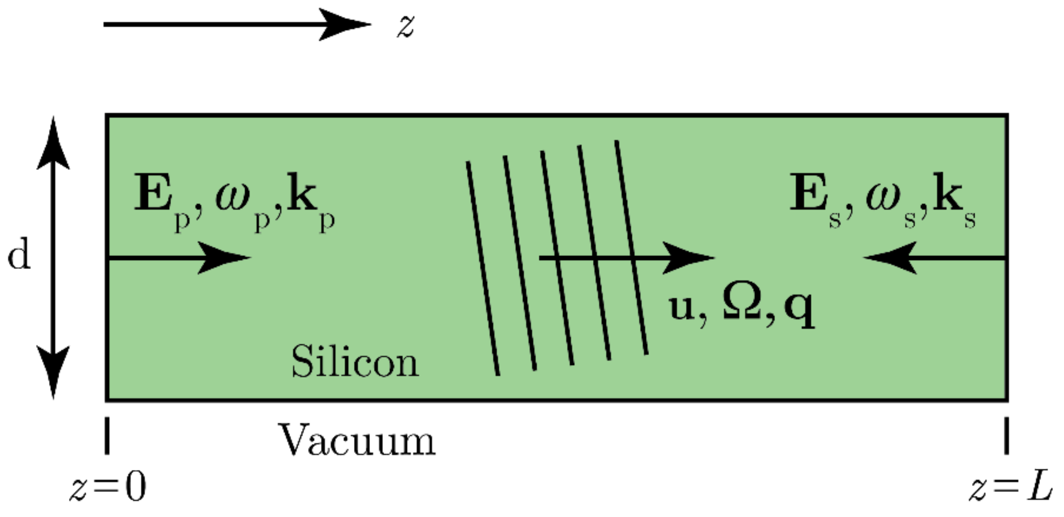
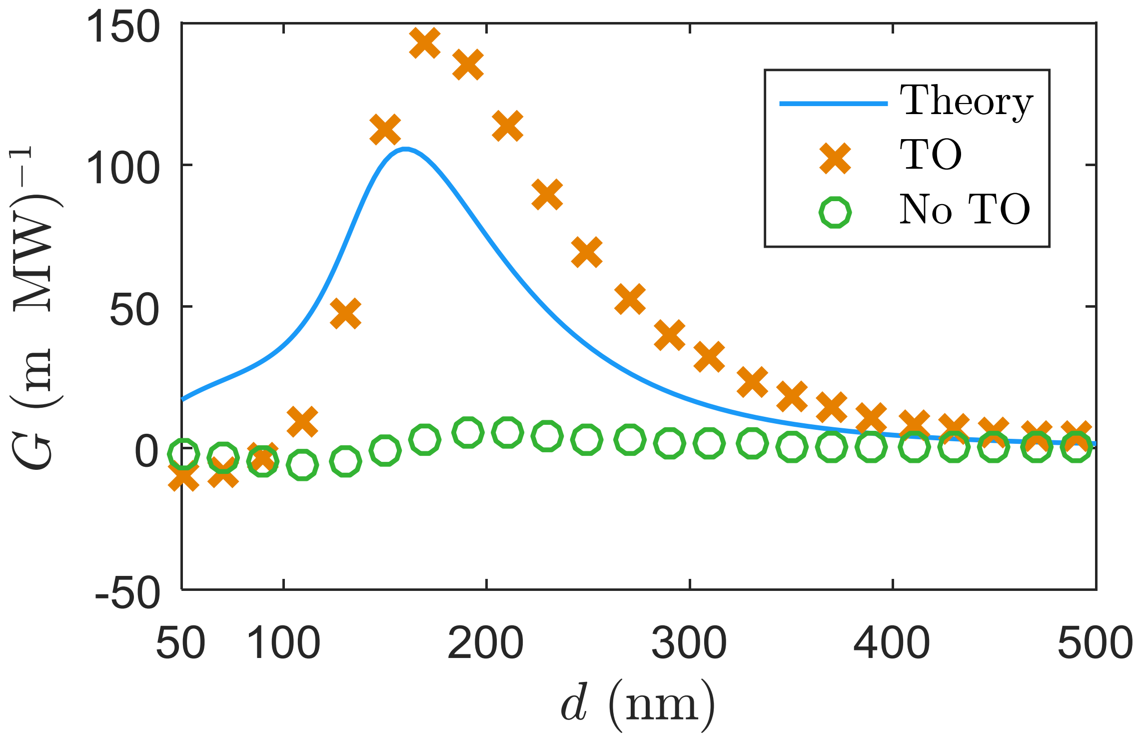
Test: dielectric slab waveguide





Theory calculations based on C. Wolff *et al., Phys. Rev. A* **92** (2015).

Conclusions and acknowledgments

- SBS in structures can be simulated using special techniques
- TO-based method can handle loss, anisotropy, complicated optical forces, solid-liquid interactions
- Future directions: application to metamaterial and plasmonic systems

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Further acknowledgments: Robert L. Bryant, Duke University

Proposed topics of discussion

- Details of COMSOL simulations
- Transformation optics
- Brillouin scattering and optical forces
- Details of method derivation
- Theoretical descriptions of SBS
- Applications

To take a closer look…

Poster session at 6:00 pm

R. Zecca *et al., Phys. Rev. A* (2016), in review

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Metric coefficients

$$\bar{\bar{g}}_0 = \bar{\bar{I}} + \frac{1}{2}\mathbb{R}\mathrm{e}\left\{(\nabla\mathbf{u})(\nabla\mathbf{u})^\dagger + \frac{\Delta\rho}{\rho_0}\left[(\nabla\mathbf{u})^* + (\nabla\mathbf{u})^\dagger\right]\right\}$$

$$\bar{\bar{g}}_1 = \frac{(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T}{2} + \frac{\Delta\rho}{2\rho_0}\left\{\bar{\bar{I}} + \frac{1}{2}\mathbb{R}\mathrm{e}\left[(\nabla\mathbf{u})(\nabla\mathbf{u})^\dagger\right]\right\} + \frac{\Delta\rho^*}{\rho_0}\frac{(\nabla\mathbf{u})(\nabla\mathbf{u})^T}{8}$$

$$\bar{\bar{g}}_2 = \frac{(\nabla\mathbf{u})(\nabla\mathbf{u})^T}{4} + \frac{\Delta\rho}{\rho_0}\frac{(\nabla\mathbf{u}) + (\nabla\mathbf{u})^T}{4}$$

$$\bar{\bar{g}}_3 = \frac{\Delta\rho}{\rho_0}\frac{(\nabla\mathbf{u})(\nabla\mathbf{u})^T}{8}$$

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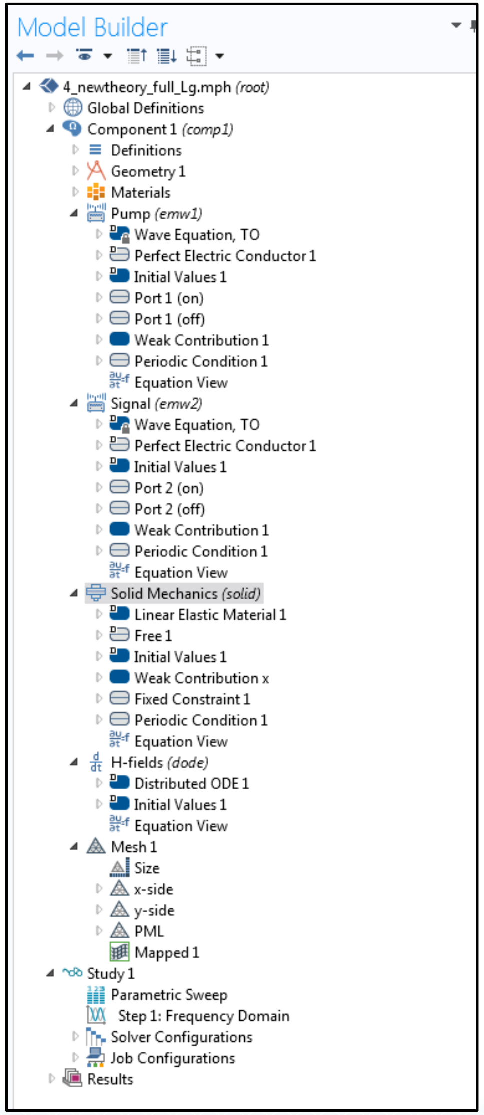
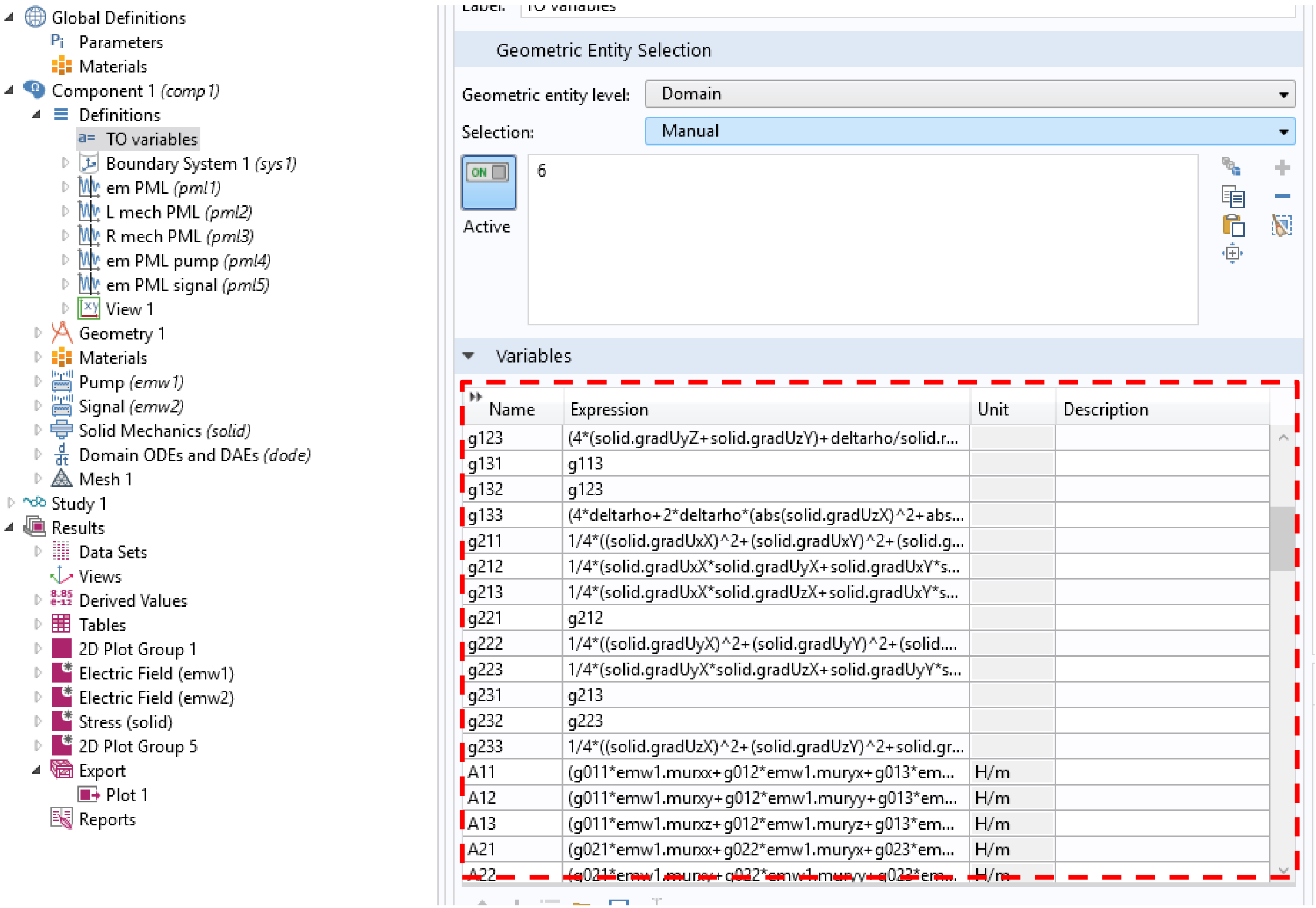
Details of COMSOL simulation

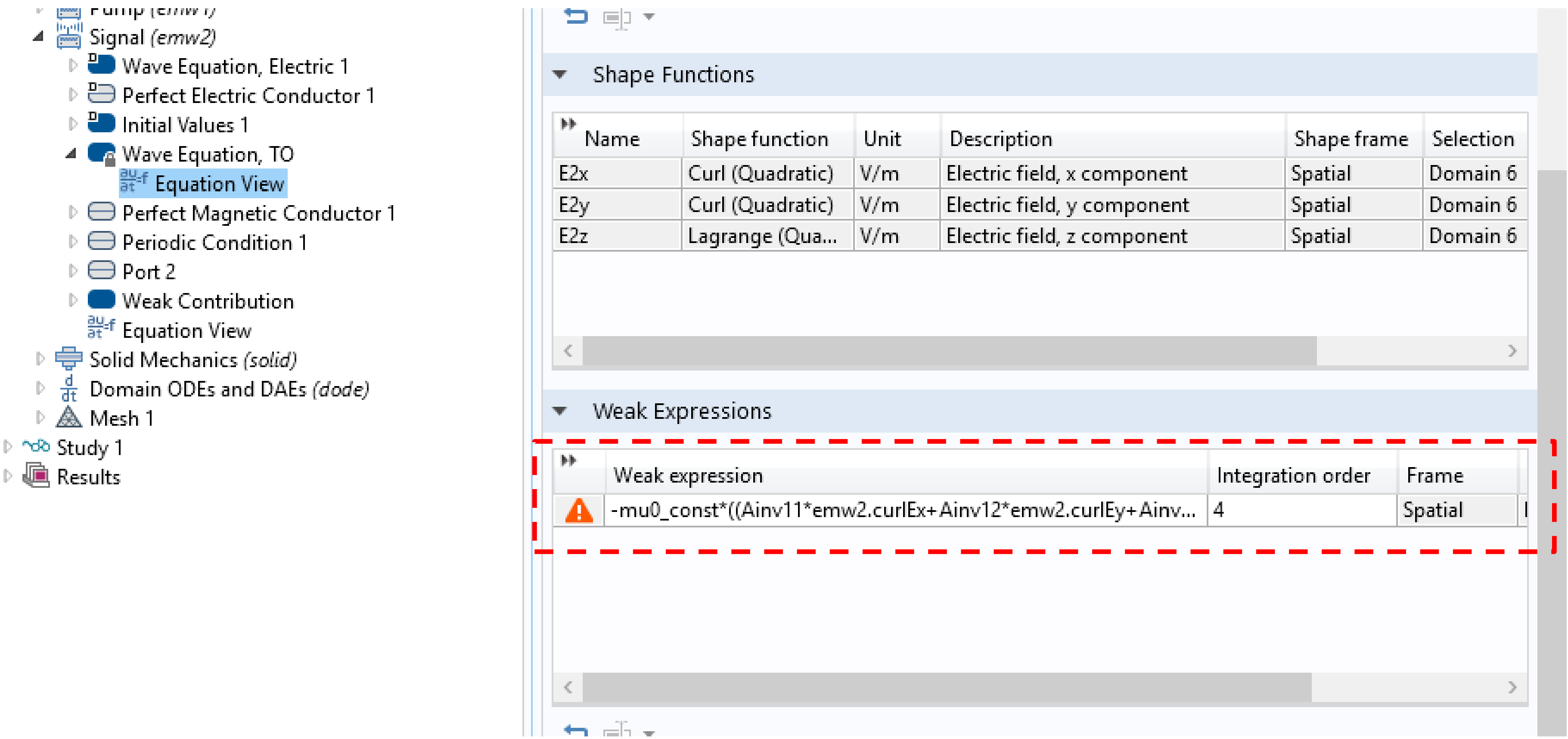


- 2 Electromagnetic Waves (RF) modules
- 1 Solid Mechanics module
- 1 Math (DODE) module (to solve for H-fields)
- Frequency “load-ramping” to ease convergence when necessary

Details of COMSOL simulation: TO metric



Details of COMSOL simulation: recasting PDEs



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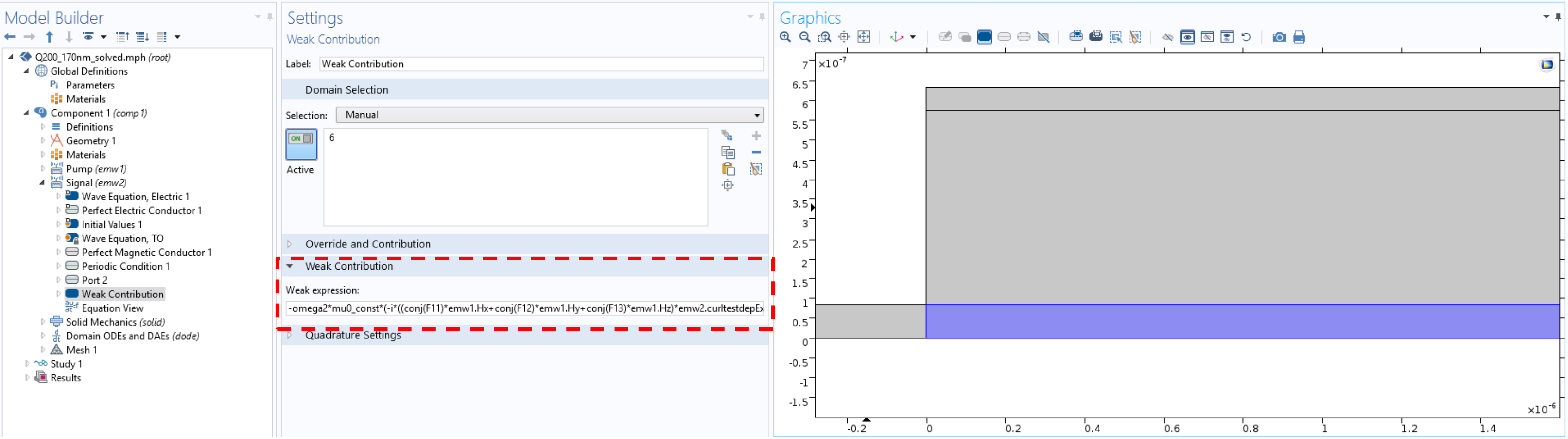
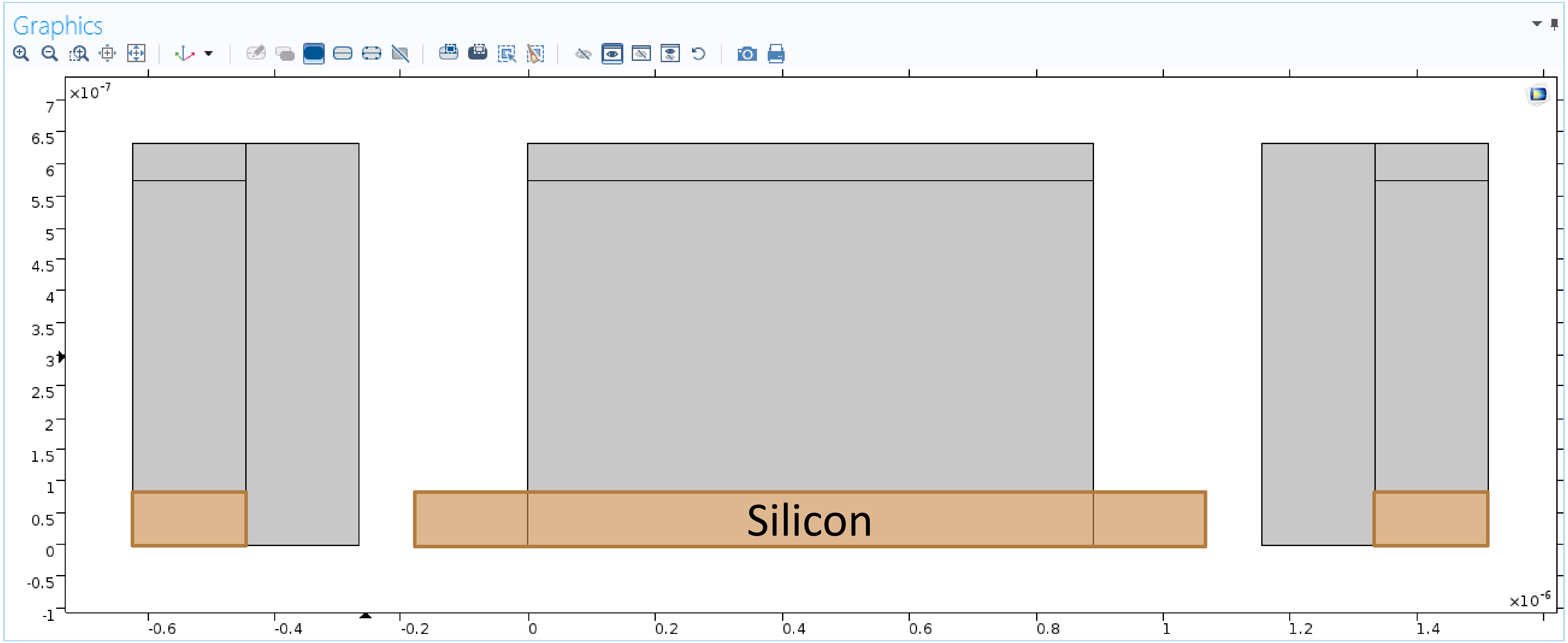
Details of COMSOL simulation: coupling



Details of COMSOL simulation: slab model



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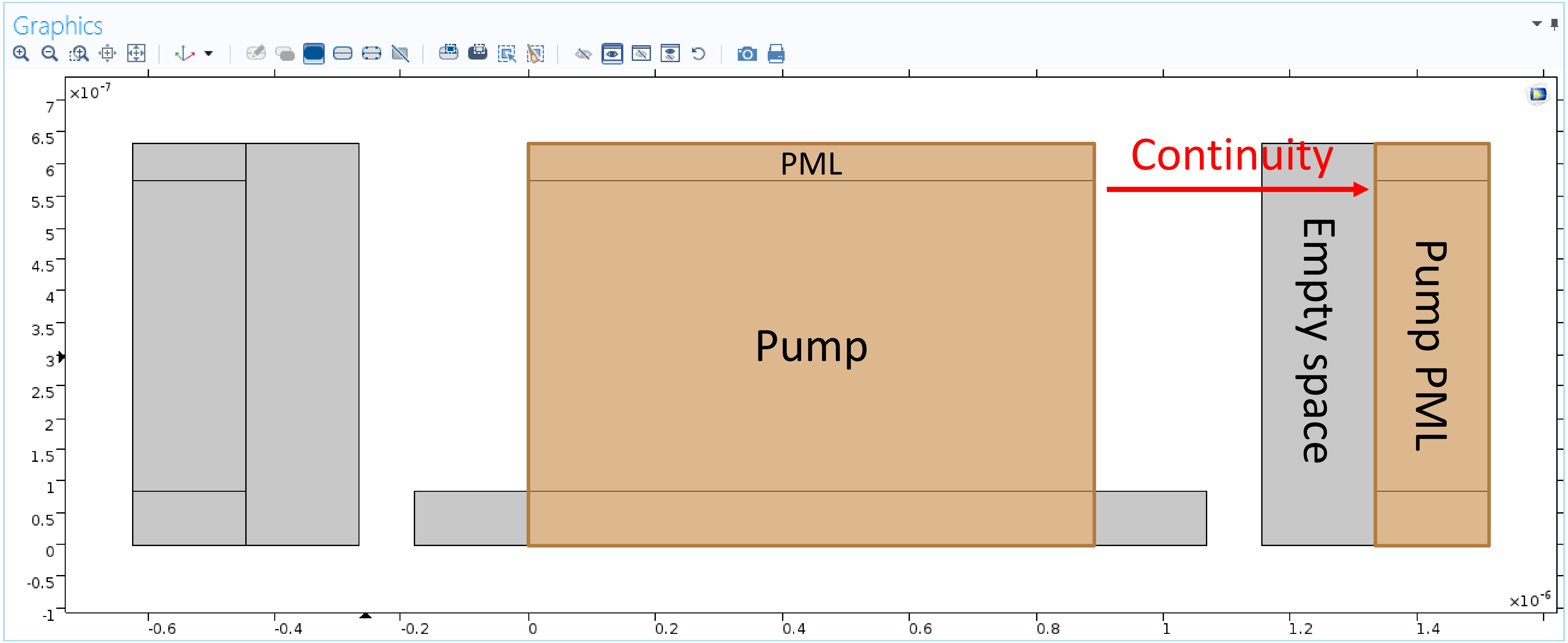

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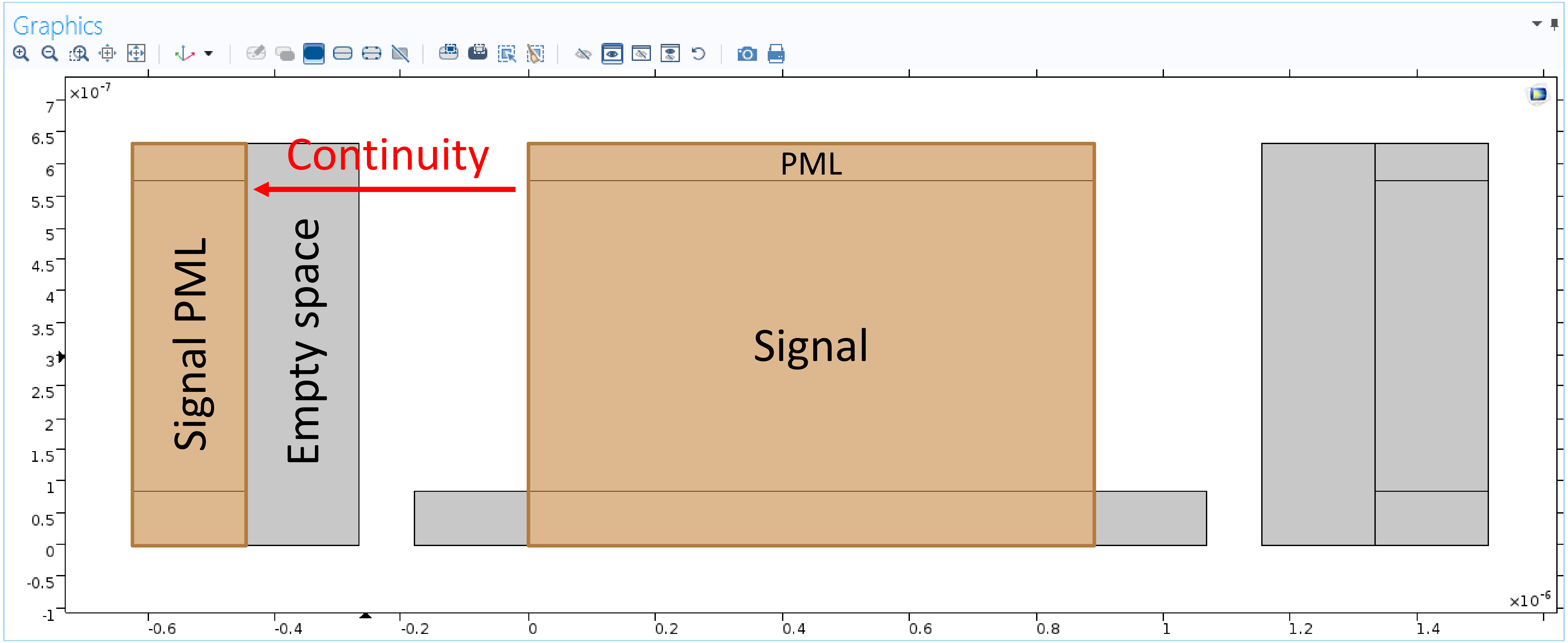

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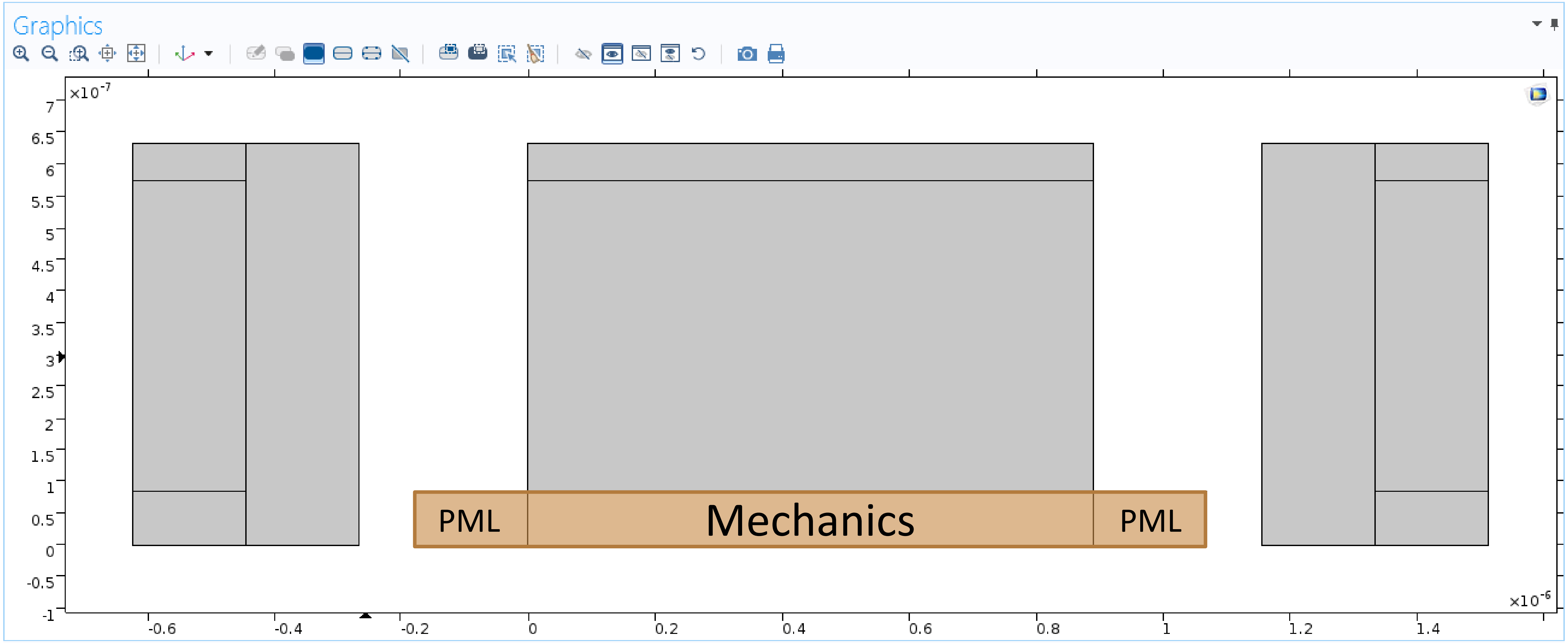
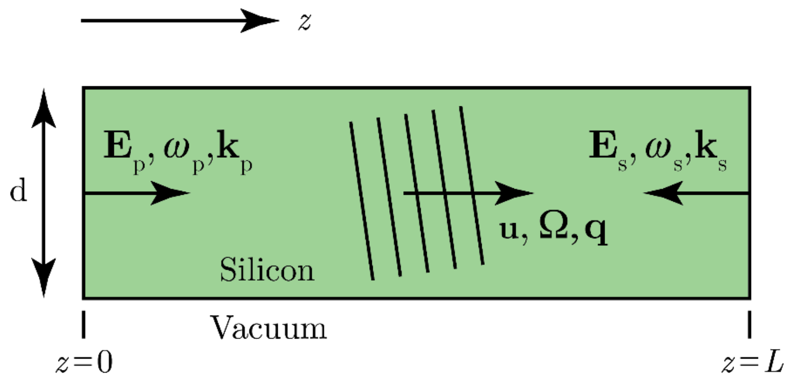
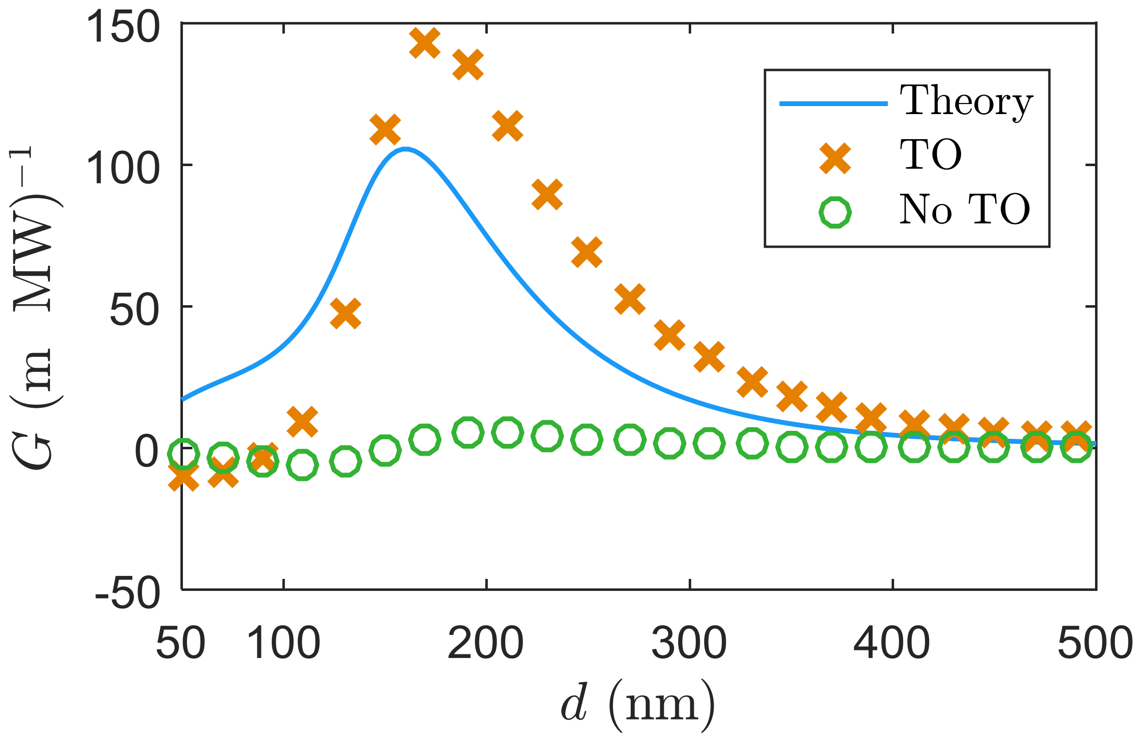


Test: dielectric slab waveguide (details)

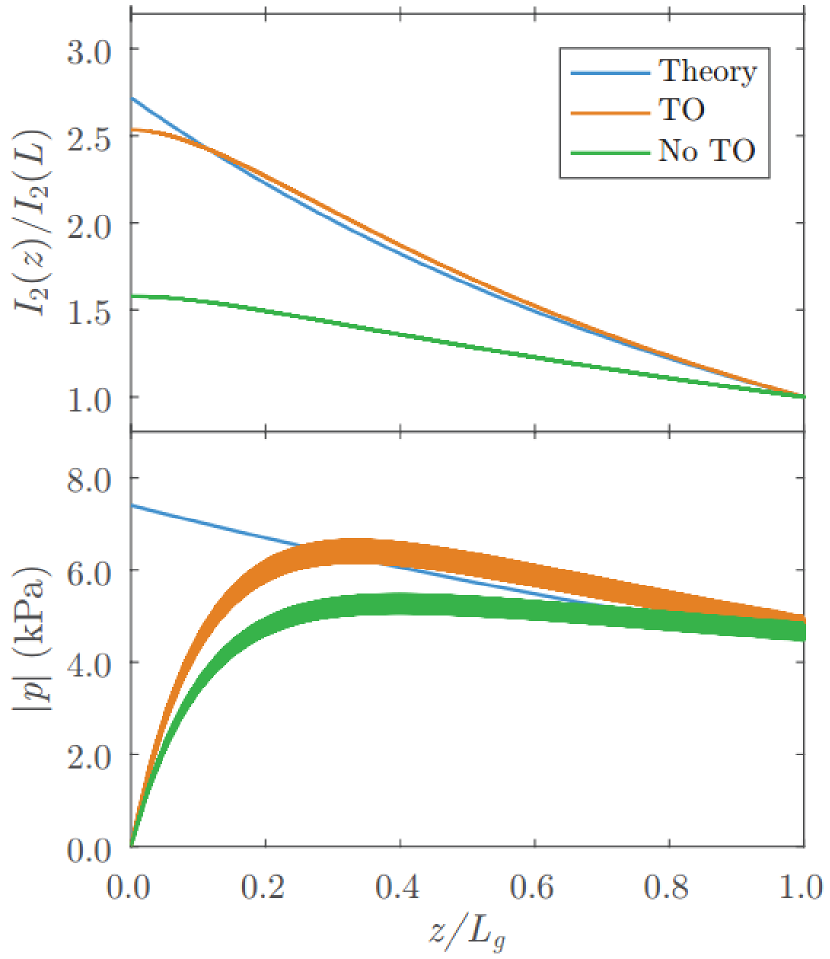


	Pump	Signal	Elastic
Field	$\mathbf{E}_p$	$\mathbf{E}_s$	$\mathbf{u}$
Frequency	ω_p	ω_s	Ω
Wavevector	$\mathbf{k}_p$	$\mathbf{k}_s$	$\mathbf{q}$
Energy cons.	$\omega_p = \omega_s + \Omega$		
Momentum cons.	$\mathbf{k}_p = \mathbf{k}_s + \mathbf{q}$		



Theory calculations based on C. Wolff *et al., Phys. Rev. A* **92** (2015).

Signal fit and gain extraction



$$I_s(z) = I_s(L)\, e^{gI_p(L-z)}$$

Theory calculations based on R. W. Boyd, *Nonlinear Optics.*

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Stress tensors

- Engineering stress tensor $\bar{\bar{\varsigma}} = \dfrac{\mathbf{F}}{A_0}$
- Cauchy stress tensor $\bar{\bar{\sigma}} = \dfrac{\mathbf{F}}{A}$
- First Piola-Kirchhoff stress tensor $\bar{\bar{P}}$
- Second Piola-Kirchhoff stress tensor $\bar{\bar{S}}$

$$\bar{\bar{S}} = \bar{\bar{F}}^{-1}\bar{\bar{P}}$$

$$\bar{\bar{\sigma}} = \bar{\bar{J}}^{-1}\bar{\bar{P}}\bar{\bar{F}}^{T} = \bar{\bar{J}}^{-1}\bar{\bar{F}}\bar{\bar{P}}\bar{\bar{F}}^{T}$$

Definitions of gain

$$\partial_z P_\mathrm{s} = G P_\mathrm{p} P_\mathrm{s}$$

Appropriate for guided systems ($\exists$ cross-section)

$$\partial_z I_\mathrm{s} = g I_\mathrm{p} I_\mathrm{s}$$

Appropriate for bulk (infinite cross-section)