



Transformation Optics Simulation Method for Stimulated Brillouin Scattering

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**COMSOL
CONFERENCE**
2016 BOSTON



**CENTER FOR METAMATERIALS
AND INTEGRATED PLASMONICS**

R. Zecca *et al.*, *Phys. Rev. A* (2016), in review



Stimulated Brillouin scattering

- Nonlinear coupling of light and elastic waves (optical forces, scattering)
- Applications: optical lasers, amplifiers, strain sensors, slow light
- Potential for high gains and SNR in (nano-)structured materials/devices
- Numerical modeling is key to device design





Finite-element SBS modeling

Stokes process: $\omega_p = \omega_s + \Omega$

MECHANICS

Displacement vectorial field

$$\tilde{\mathbf{u}} = \text{Re} (\mathbf{u} e^{i\Omega t})$$

Mass density variation scalar field

$$\Delta \tilde{\rho} = \text{Re} (\Delta \rho e^{i\Omega t})$$

$$\Delta \rho \propto \nabla \mathbf{u}$$

OPTICS

Bi-chromatic electromagnetic field

$$\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_p + \tilde{\mathbf{E}}_s$$

$$\tilde{\mathbf{H}} = \tilde{\mathbf{H}}_p + \tilde{\mathbf{H}}_s$$

$$\tilde{\mathbf{E}}_n = \text{Re} (\mathbf{E}_n e^{i\omega_n t})$$

$$\tilde{\mathbf{H}}_n = \text{Re} (\mathbf{H}_n e^{i\omega_n t}) \quad n = p, s$$



Finite-element SBS modeling

MECHANICS

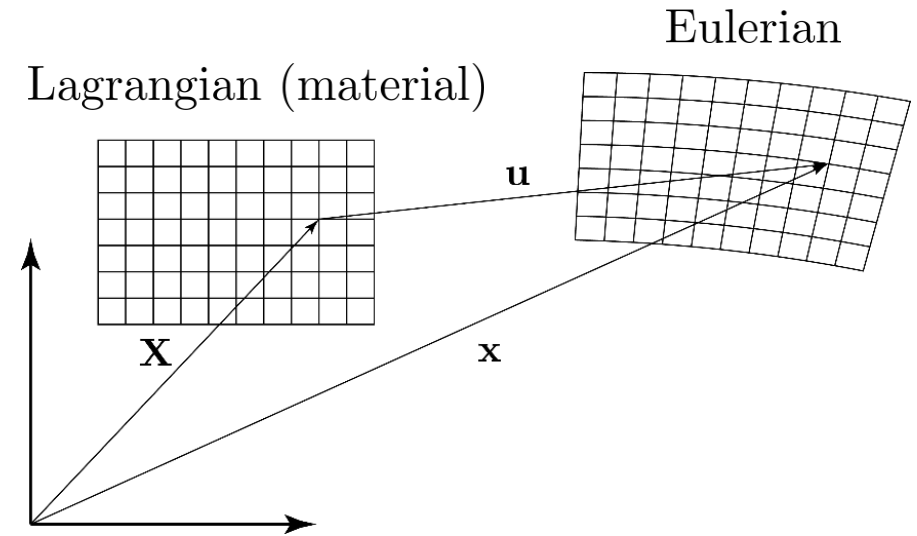
$$\begin{array}{|c|} \hline \text{mass x acceleration} \\ \hline -\rho_0 \Omega^2 \mathbf{u} \\ \hline \end{array} = \begin{array}{|c|} \hline \text{internal force (stress)} \\ \hline \nabla_{\mathbf{X}} \cdot \left(\bar{\bar{F}} \bar{\bar{S}} \right) \\ \hline \end{array} + \begin{array}{|c|} \hline \text{optical forces (external)} \\ \hline \mathbf{f} \left(\nabla_{\mathbf{X}} \left(\mathbf{E}_p \cdot \mathbf{E}_s^* \right) \right) \\ \hline \end{array}$$

OPTICS

$$\begin{aligned}
 \nabla^2 \mathbf{E}_p + k_p^2 \mathbf{E}_p &= C_1 \Delta \epsilon(\mathbf{u}) \mathbf{E}_s \\
 \nabla^2 \mathbf{E}_s + k_s^2 \mathbf{E}_s &= C_2 \Delta \epsilon^*(\mathbf{u}) \mathbf{E}_p
 \end{aligned}$$

$$d\mathbf{x} = \bar{\bar{F}} d\mathbf{X}$$

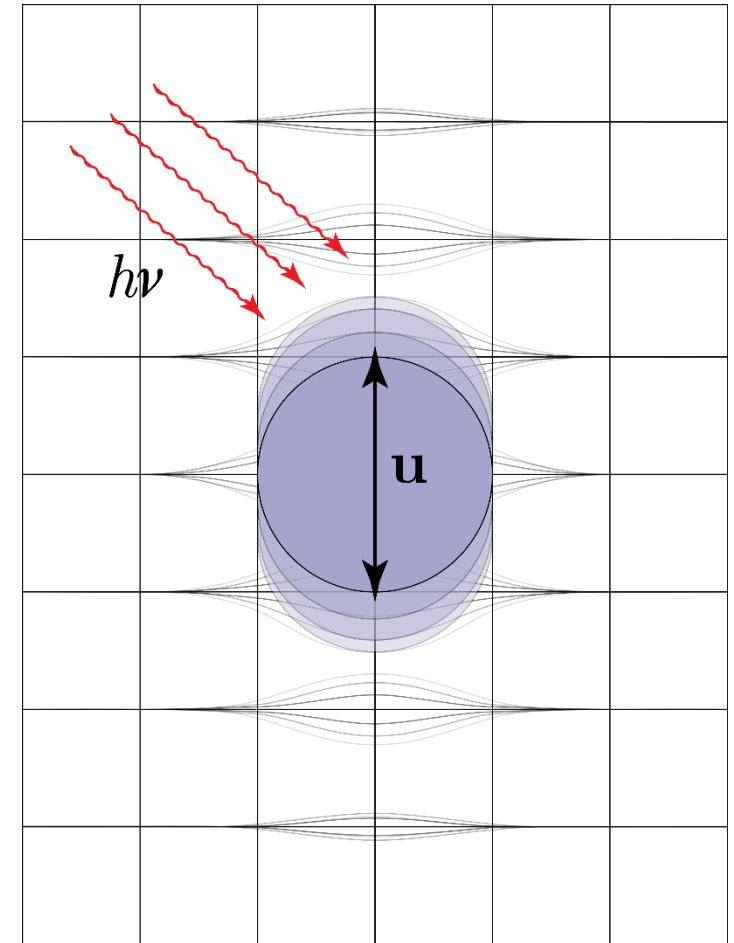
$$\bar{\bar{F}} = \bar{\bar{F}}(\mathbf{u}, \Delta \rho)$$





Finite-element SBS modeling

- Frequency-domain EM solver cannot take moving geometry into account
- Significant problem in the case of SBS
- Time-domain too onerous: $\frac{2\pi}{\Omega} \gg \frac{2\pi}{\omega}$ (10 ps vs. fs)
- Different strategy needed





Transformation optics: designing an invisibility cloak

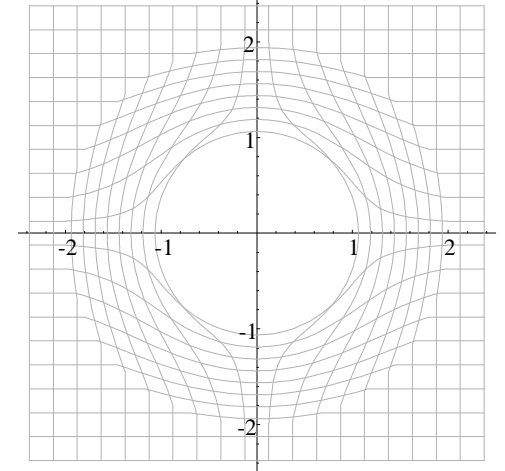
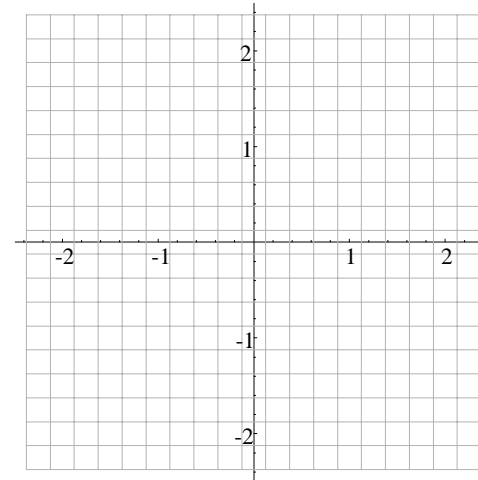
Given a coordinate transform

$$\mathbf{x} \rightarrow \mathbf{x}' \quad \bar{\Lambda} = \frac{\partial \mathbf{x}'}{\partial \mathbf{x}}$$

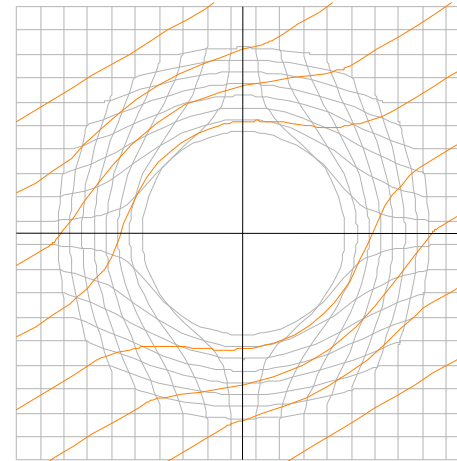
Using form-invariance of Maxwell's equations

$$\bar{\epsilon}' = \frac{\bar{\Lambda} \bar{\epsilon} \bar{\Lambda}^T}{\det \bar{\Lambda}}$$

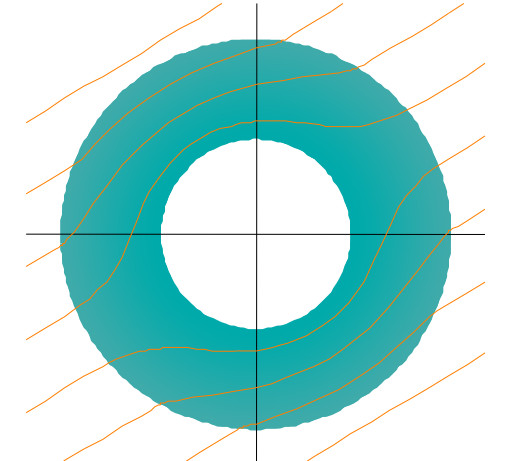
$$\bar{\mu}' = \frac{\bar{\Lambda} \bar{\mu} \bar{\Lambda}^T}{\det \bar{\Lambda}}$$



Topological



Material





Time-dependent TO metric

- Represent movement by equivalent, time-dependent material properties
- Apply time-dependent metric, using deformation gradient $\bar{\bar{F}}$ as Jacobian

$$\bar{\bar{\epsilon}}' = \bar{\bar{g}} \epsilon$$

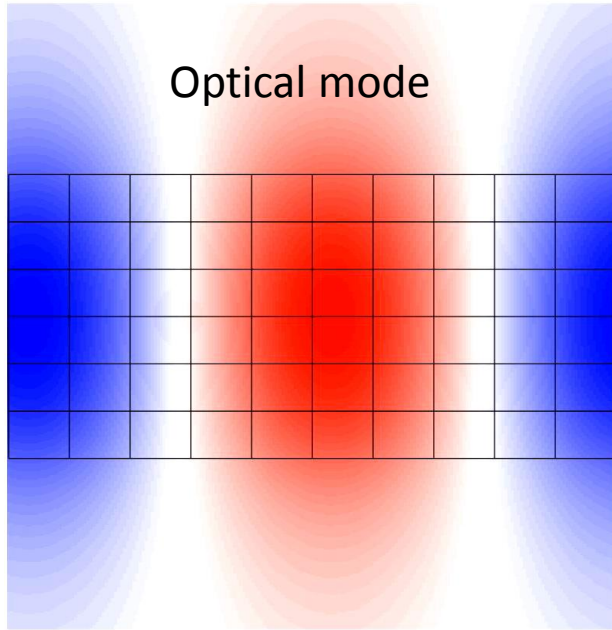
$$\bar{\bar{\mu}}' = \bar{\bar{g}} \mu$$

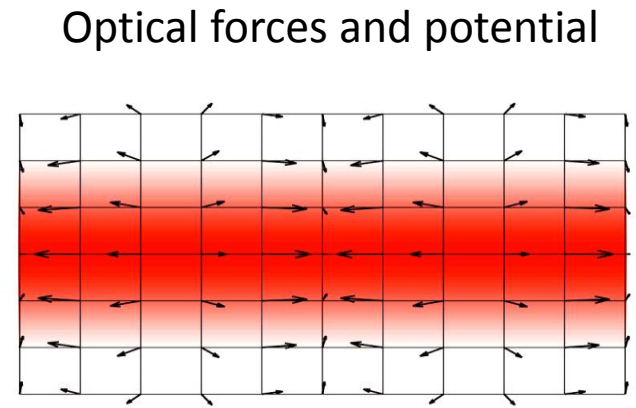
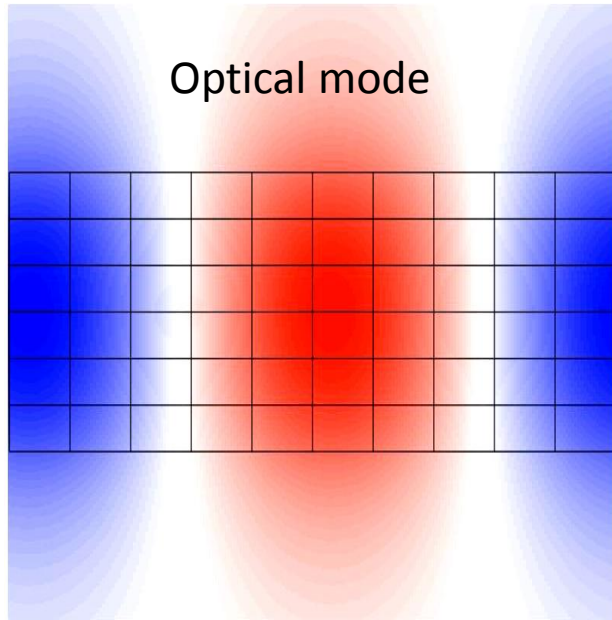
$$\bar{\bar{g}} = \frac{\bar{\bar{F}} \bar{\bar{F}}^T}{\det \bar{\bar{F}}}$$

$$\bar{\bar{g}} = \sum_{n=-3}^3 \bar{\bar{g}}_n e^{in\Omega t}$$

$$\bar{\bar{g}}_n = \bar{\bar{g}}_{-n}^*$$

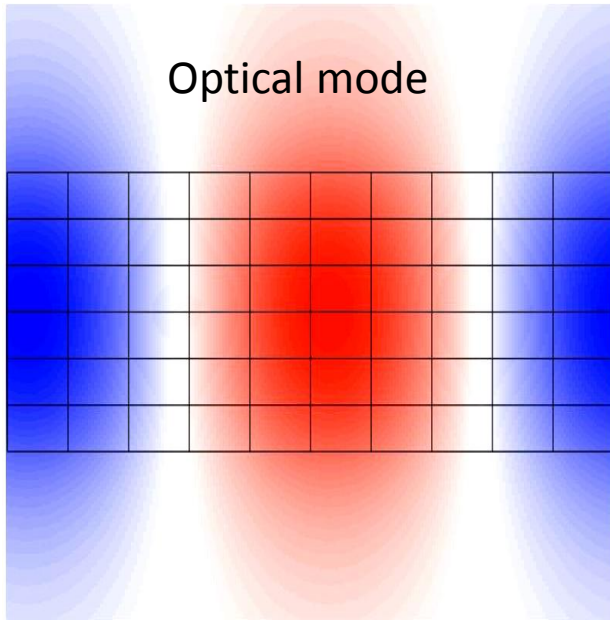
$$\bar{\bar{g}} \rightarrow \bar{\bar{I}} \quad \text{for} \quad \mathbf{u} \rightarrow \mathbf{0}$$



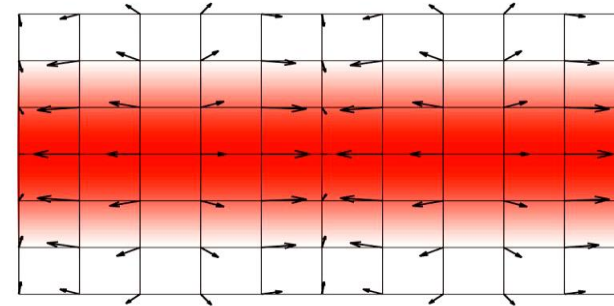




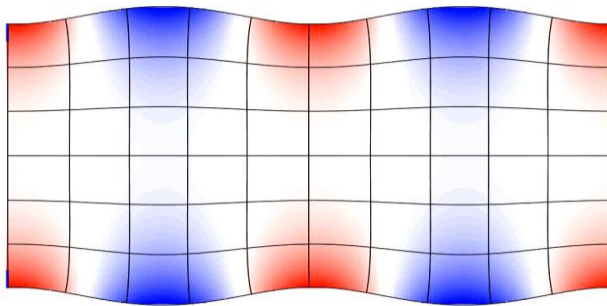
Optical mode



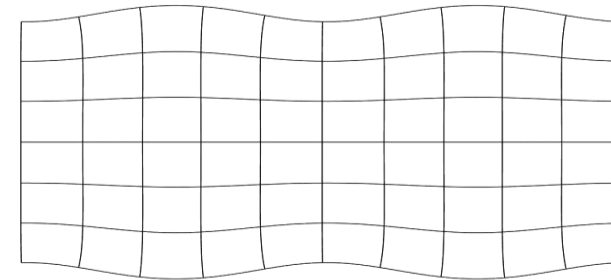
Optical forces and potential



Mass density/permittivity variation



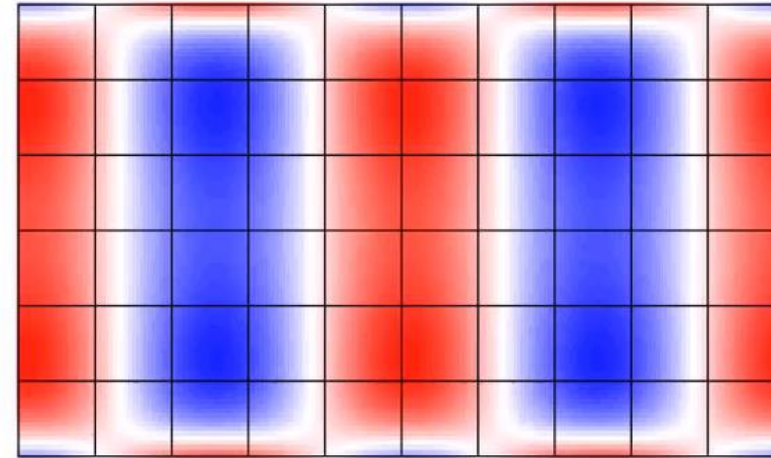
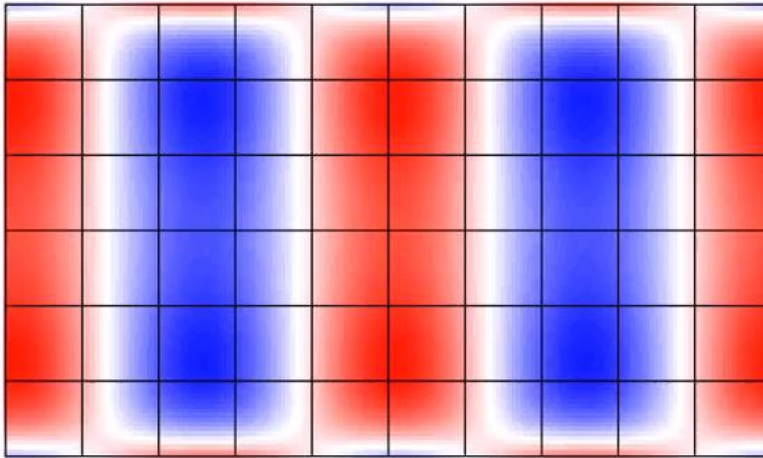
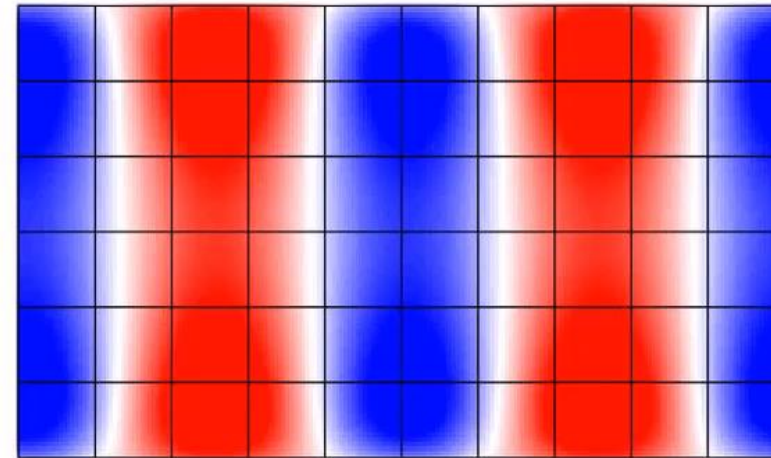
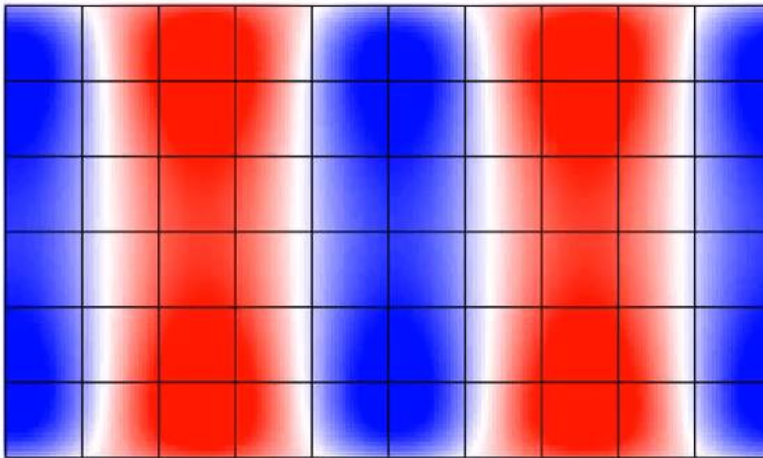
Permeability variation





$$\frac{\epsilon_r + \Delta\epsilon}{\epsilon_r}$$

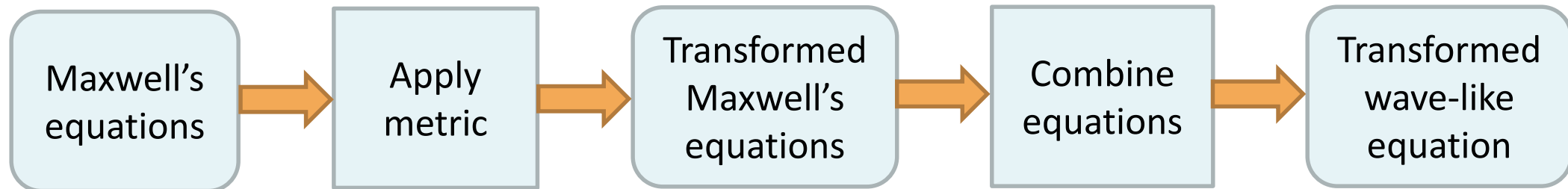
$$\frac{\mu_r + \Delta\mu}{\mu_r}$$

Longitudinal
componentsTransverse
components



Transformed Maxwell's equations

- COMSOL master equation $\nabla \times (\bar{\bar{\mu}}_r^{-1} \nabla \times \mathbf{E}) - k_0^2 \left(\bar{\bar{\epsilon}}_r - \frac{i\bar{\bar{\sigma}}_e}{\omega\epsilon_0} \right) \mathbf{E} = \mathbf{0}$
- Apply TO metric to Maxwell's equations and obtain similar expression



Transformed Maxwell's equations

- COMSOL master equation $\nabla \times (\bar{\bar{\mu}}_r^{-1} \nabla \times \mathbf{E}) - k_0^2 \left(\bar{\bar{\epsilon}}_r - \frac{i\bar{\bar{\sigma}}_e}{\omega\epsilon_0} \right) \mathbf{E} = \mathbf{0}$
- Apply TO metric to Maxwell's equations and obtain similar expression

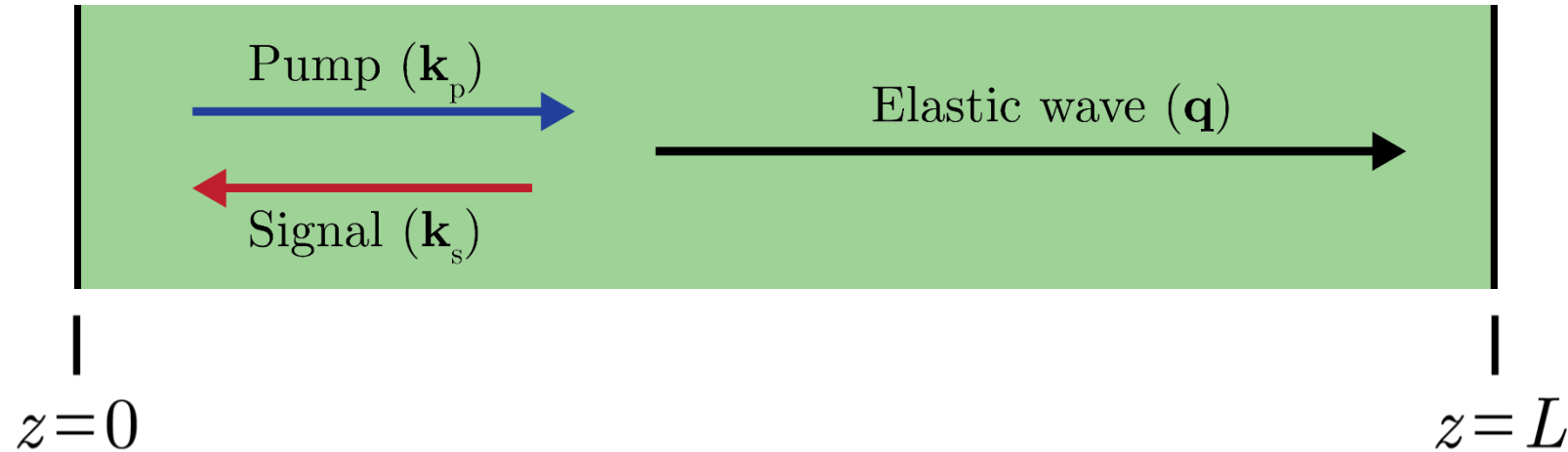
$$\nabla \times \bar{\bar{A}}^{-1} \nabla \times \mathbf{E}_p - \omega_p^2 \bar{\bar{C}} \mathbf{E}_p = \omega_p^2 \left[a \bar{\bar{K}} \mathbf{E}_p + \left(\bar{\bar{D}} + a \bar{\bar{L}} \right) \mathbf{E}_s \right] - i\omega_p \nabla \times \left(\bar{\bar{A}}^{-1} \bar{\bar{B}} \mathbf{H}_s \right)$$

$$\nabla \times \bar{\bar{A}}^{-1} \nabla \times \mathbf{E}_s - \omega_s^2 \bar{\bar{C}} \mathbf{E}_s = \omega_s^2 \left[\left(\bar{\bar{D}}^* + a \bar{\bar{L}}^* \right) \mathbf{E}_p + a \bar{\bar{K}} \mathbf{E}_s \right] - i\omega_s \nabla \times \left(\bar{\bar{A}}^{-1} \bar{\bar{B}}^* \mathbf{H}_p \right)$$

$$\bar{\bar{A}} = \bar{g}_0 \mu \quad \bar{\bar{B}} = \bar{g}_1 \mu \quad \bar{\bar{C}} = \bar{g}_0 \epsilon \quad \bar{\bar{D}} = \bar{g}_1 \epsilon \quad \bar{\bar{K}} = \epsilon_0 (\bar{g}_1 \Delta \epsilon^* + \bar{g}_1^* \Delta \epsilon) / 2 \quad \bar{\bar{L}} = \epsilon_0 (\bar{g}_0 \Delta \epsilon + \bar{g}_2 \Delta \epsilon^*) / 2$$



Test: 1D bulk amplifier



Theory

Undepleted pump approximation

$$I_s(z) = I_s(L) e^{gI_p(L-z)}$$

R. W. Boyd, *Nonlinear Optics*.

TO method

Modified wave-like equations

$$\bar{\bar{g}} = \sum_{n=-3}^3 \bar{\bar{g}}_n e^{in\Omega t}$$

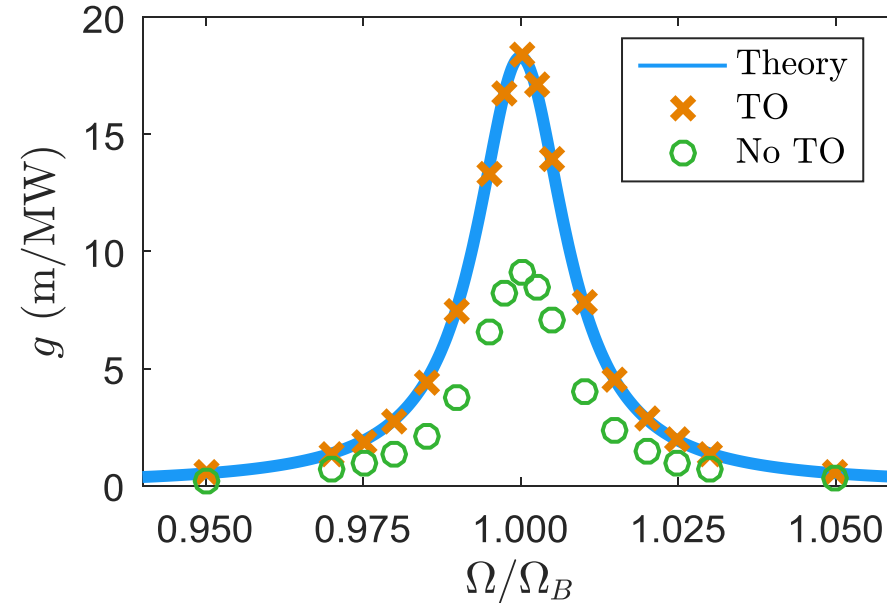
Simple coupling

Standard wave-like equations

$$\bar{\bar{g}} = \bar{\bar{I}}$$



Test: 1D bulk amplifier



Theory

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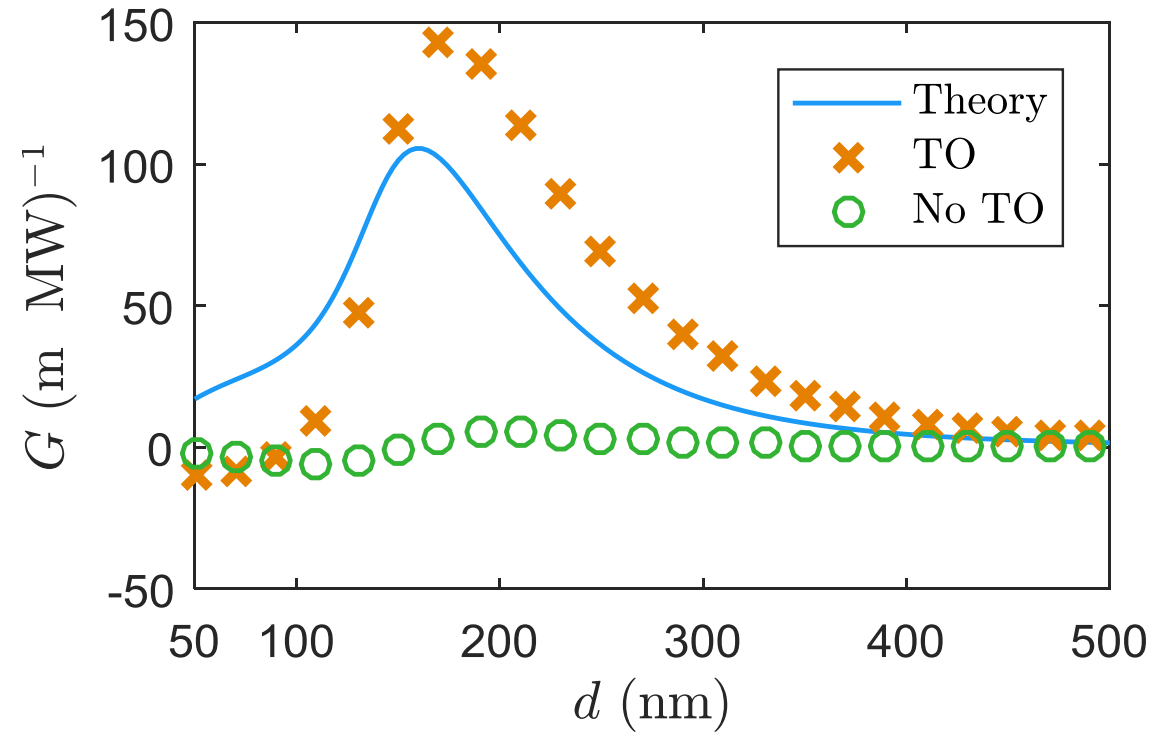
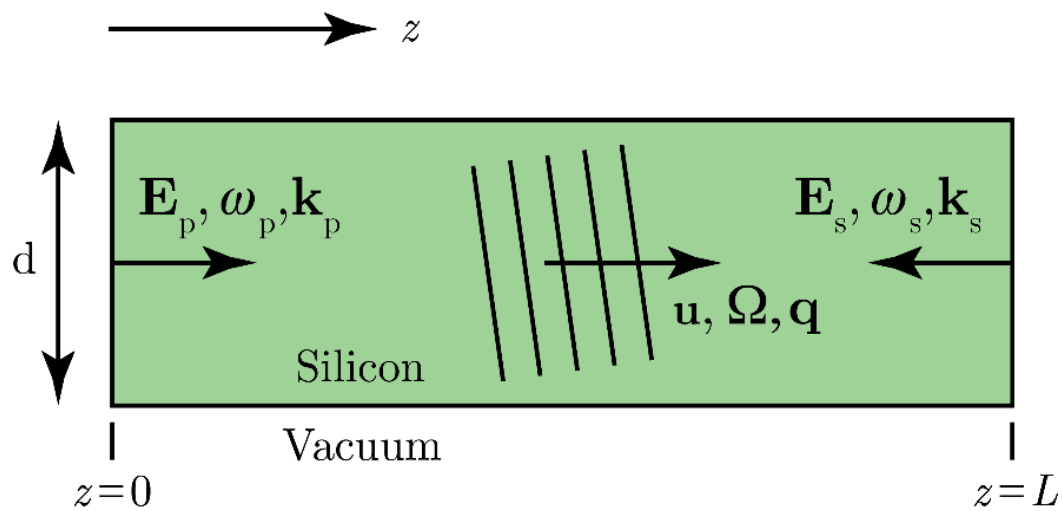
Simple coupling

Standard wave-like equations

$$\bar{\bar{g}} = \bar{\bar{I}}$$



Test: dielectric slab waveguide



Theory calculations based on C. Wolff *et al.*, *Phys. Rev. A* **92** (2015).

Conclusions and acknowledgments

- SBS in structures can be simulated using special techniques
- TO-based method can handle loss, anisotropy, complicated optical forces, solid-liquid interactions
- Future directions: application to metamaterial and plasmonic systems



Further acknowledgments: Robert L. Bryant, Duke University



Proposed topics of discussion

- Details of COMSOL simulations
 - Transformation optics
 - Brillouin scattering and optical forces
 - Details of method derivation
 - Theoretical descriptions of SBS
 - Applications
-

To take a closer look...

Poster session at 6:00 pm

R. Zecca *et al.*, *Phys. Rev. A* (2016),
in review

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Metric coefficients

$$\bar{g}_0 = \bar{I} + \frac{1}{2} \text{Re} \left\{ (\nabla \mathbf{u}) (\nabla \mathbf{u})^\dagger + \frac{\Delta \rho}{\rho_0} \left[(\nabla \mathbf{u})^* + (\nabla \mathbf{u})^\dagger \right] \right\}$$

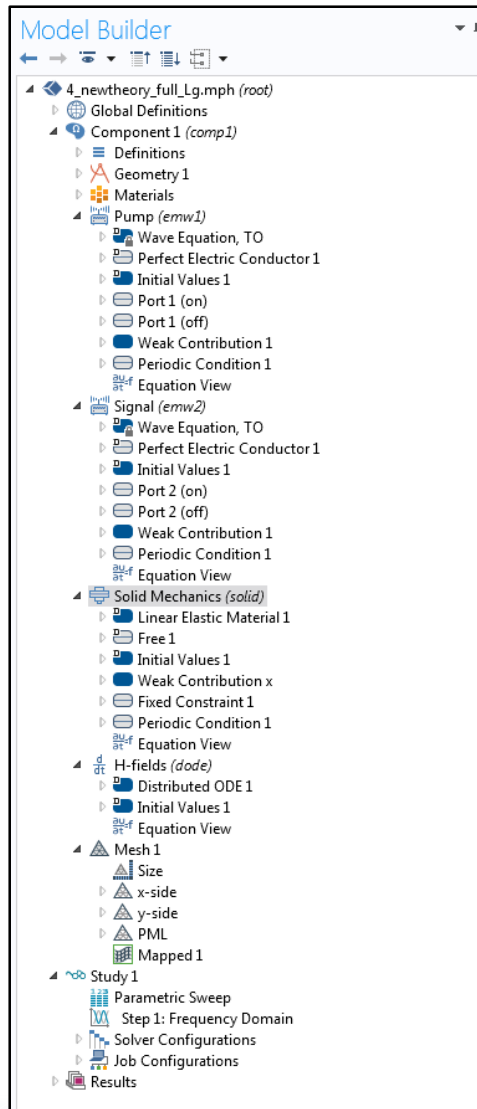
$$\bar{g}_1 = \frac{(\nabla \mathbf{u}) + (\nabla \mathbf{u})^T}{2} + \frac{\Delta \rho}{2\rho_0} \left\{ \bar{I} + \frac{1}{2} \text{Re} \left[(\nabla \mathbf{u}) (\nabla \mathbf{u})^\dagger \right] \right\} + \frac{\Delta \rho^*}{\rho_0} \frac{(\nabla \mathbf{u}) (\nabla \mathbf{u})^T}{8}$$

$$\bar{g}_2 = \frac{(\nabla \mathbf{u}) (\nabla \mathbf{u})^T}{4} + \frac{\Delta \rho}{\rho_0} \frac{(\nabla \mathbf{u}) + (\nabla \mathbf{u})^T}{4}$$

$$\bar{g}_3 = \frac{\Delta \rho}{\rho_0} \frac{(\nabla \mathbf{u}) (\nabla \mathbf{u})^T}{8}$$



Details of COMSOL simulation



- 2 Electromagnetic Waves (RF) modules
- 1 Solid Mechanics module
- 1 Math (DODE) module (to solve for H-fields)
- Frequency “load-ramping” to ease convergence when necessary

Details of COMSOL simulation: TO metric

- Global Definitions
 - Parameters
 - Materials
- Component 1 (comp 1)
 - Definitions
 - TO variables
 - Boundary System 1 (sys 1)
 - em PML (pml1)
 - L mech PML (pml2)
 - R mech PML (pml3)
 - em PML pump (pml4)
 - em PML signal (pml5)
 - View 1
 - Geometry 1
 - Materials
 - Pump (emw1)
 - Signal (emw2)
 - Solid Mechanics (solid)
 - Domain ODEs and DAEs (dode)
 - Mesh 1
- Study 1
- Results
 - Data Sets
 - Views
 - Derived Values
 - Tables
 - 2D Plot Group 1
 - Electric Field (emw1)
 - Electric Field (emw2)
 - Stress (solid)
 - 2D Plot Group 5
 - Export
 - Plot 1
 - Reports

Level: TO variables

Geometric Entity Selection

Geometric entity level: Domain

Selection: Manual

6

Active

Variables

Name	Expression	Unit	Description
g123	$(4*(\text{solid.gradUyZ}+\text{solid.gradUzY})+\text{deltarho}/\text{solid.r...}$		
g131	g113		
g132	g123		
g133	$(4*\text{deltarho}+2*\text{deltarho}*(\text{abs}(\text{solid.gradUzX})^2+\text{abs}...$		
g211	$1/4*((\text{solid.gradUxX})^2+(\text{solid.gradUxY})^2+(\text{solid.g...}$		
g212	$1/4*(\text{solid.gradUxX}*\text{solid.gradUyX}+\text{solid.gradUxY}*s...$		
g213	$1/4*(\text{solid.gradUxX}*\text{solid.gradUzX}+\text{solid.gradUxY}*s...$		
g221	g212		
g222	$1/4*((\text{solid.gradUyX})^2+(\text{solid.gradUyY})^2+(\text{solid....}$		
g223	$1/4*(\text{solid.gradUyX}*\text{solid.gradUzX}+\text{solid.gradUyY}*s...$		
g231	g213		
g232	g223		
g233	$1/4*((\text{solid.gradUzX})^2+(\text{solid.gradUzY})^2+\text{solid.gr...}$		
A11	$(\text{g011}*\text{emw1.murxx}+\text{g012}*\text{emw1.muryx}+\text{g013}*\text{em...}$	H/m	
A12	$(\text{g011}*\text{emw1.murxy}+\text{g012}*\text{emw1.muryy}+\text{g013}*\text{em...}$	H/m	
A13	$(\text{g011}*\text{emw1.murxz}+\text{g012}*\text{emw1.muryz}+\text{g013}*\text{em...}$	H/m	
A21	$(\text{g021}*\text{emw1.murxx}+\text{g022}*\text{emw1.muryx}+\text{g023}*\text{em...}$	H/m	
A22	$(\text{g021}*\text{emw1.murxy}+\text{g022}*\text{emw1.muryy}+\text{g023}*\text{em...}$	H/m	


Details of COMSOL simulation: recasting PDEs

- ▶ Pump (emw1)
- ▶ Signal (emw2)
 - ▶ Wave Equation, Electric 1
 - ▶ Perfect Electric Conductor 1
 - ▶ Initial Values 1
 - ▶ Wave Equation, TO
 - ▶ **Equation View**
 - ▶ Perfect Magnetic Conductor 1
 - ▶ Periodic Condition 1
 - ▶ Port 2
 - ▶ Weak Contribution
 - ▶ Equation View
 - ▶ Solid Mechanics (solid)
 - ▶ Domain ODEs and DAEs (dode)
 - ▶ Mesh 1
- ▶ Study 1
- ▶ Results

Shape Functions

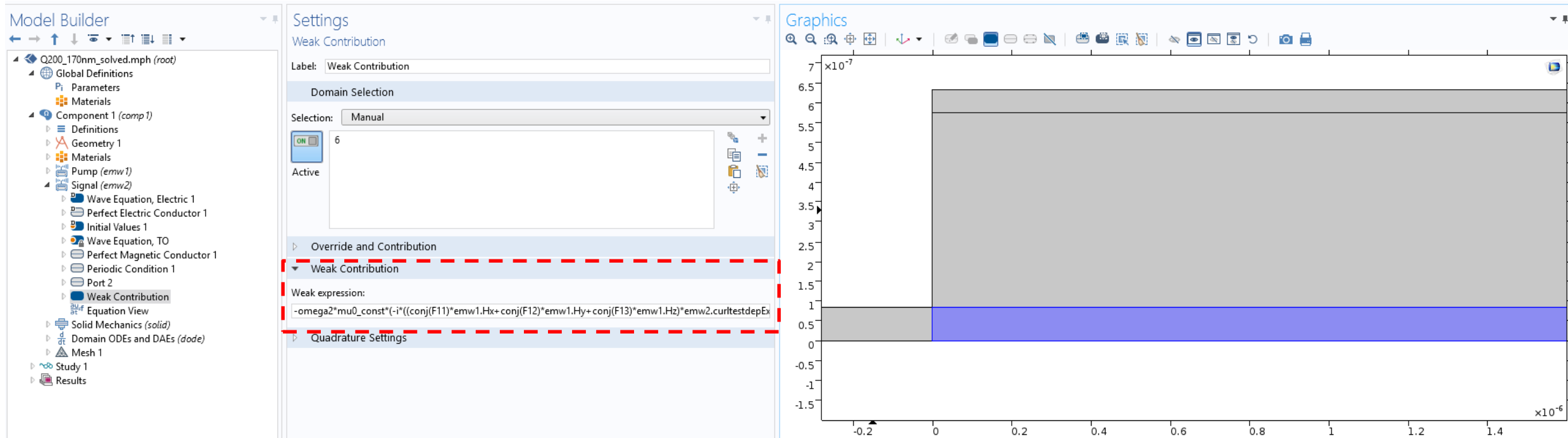
Name	Shape function	Unit	Description	Shape frame	Selection
E2x	Curl (Quadratic)	V/m	Electric field, x component	Spatial	Domain 6
E2y	Curl (Quadratic)	V/m	Electric field, y component	Spatial	Domain 6
E2z	Lagrange (Qua...	V/m	Electric field, z component	Spatial	Domain 6

Weak Expressions

Weak expression	Integration order	Frame
 $-\mu_0_{\text{const}}*((\text{Ainv11}*\text{emw2}.\text{curlEx}+\text{Ainv12}*\text{emw2}.\text{curlEy}+\text{Ainv}...$	4	Spatial

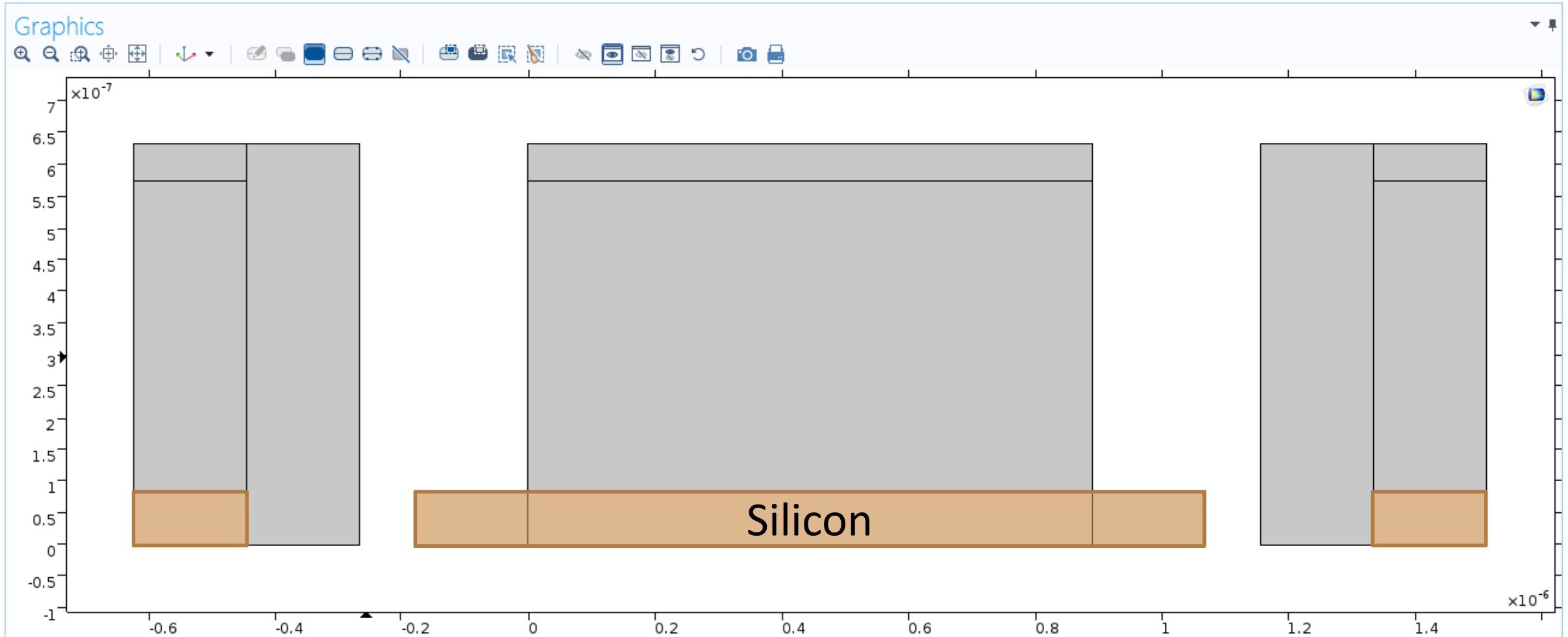


Details of COMSOL simulation: coupling



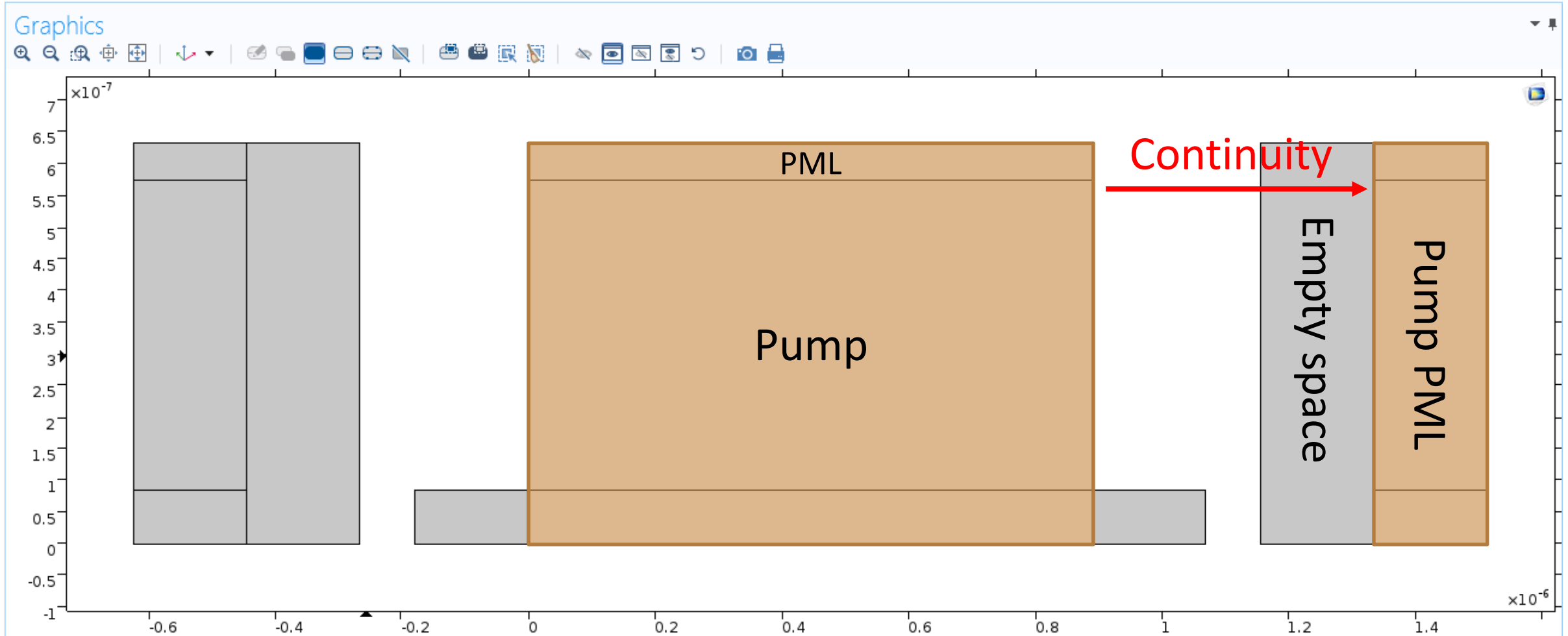


Details of COMSOL simulation: slab model



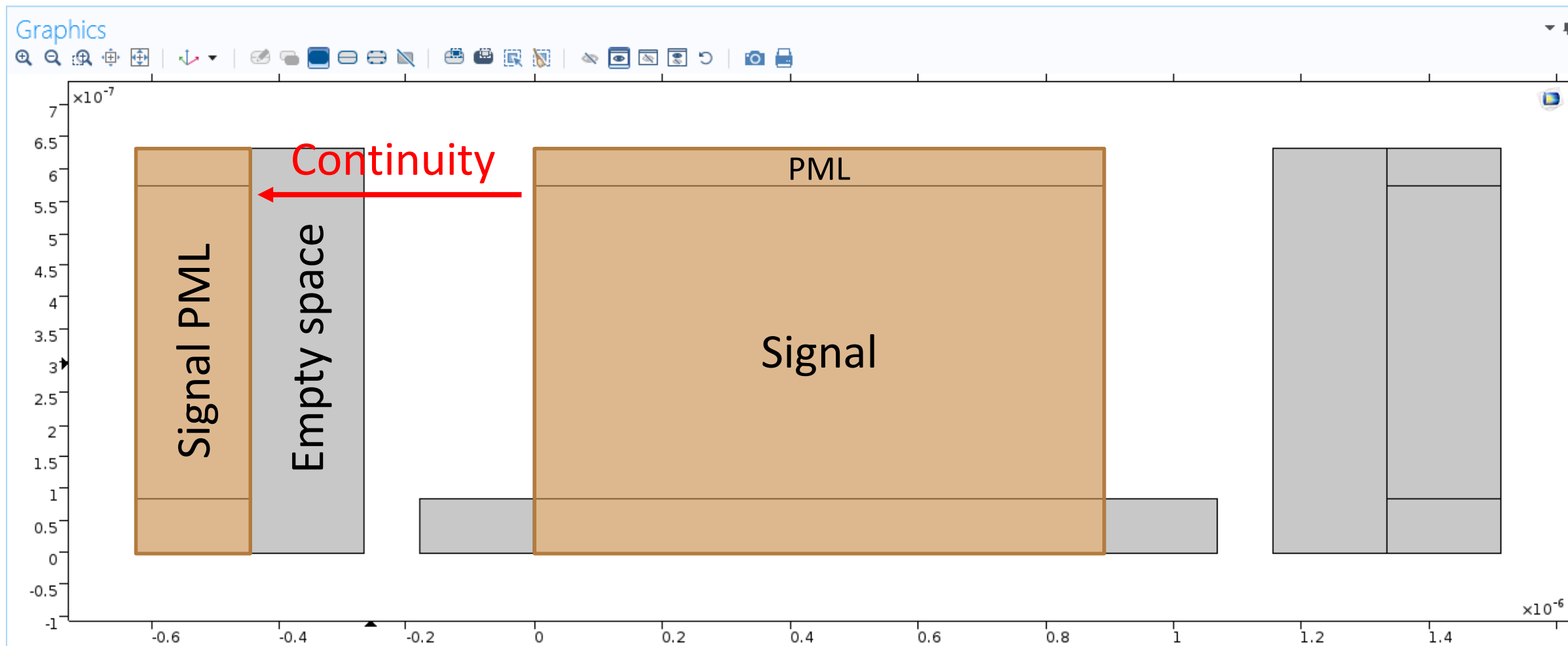


Details of COMSOL simulation: slab model



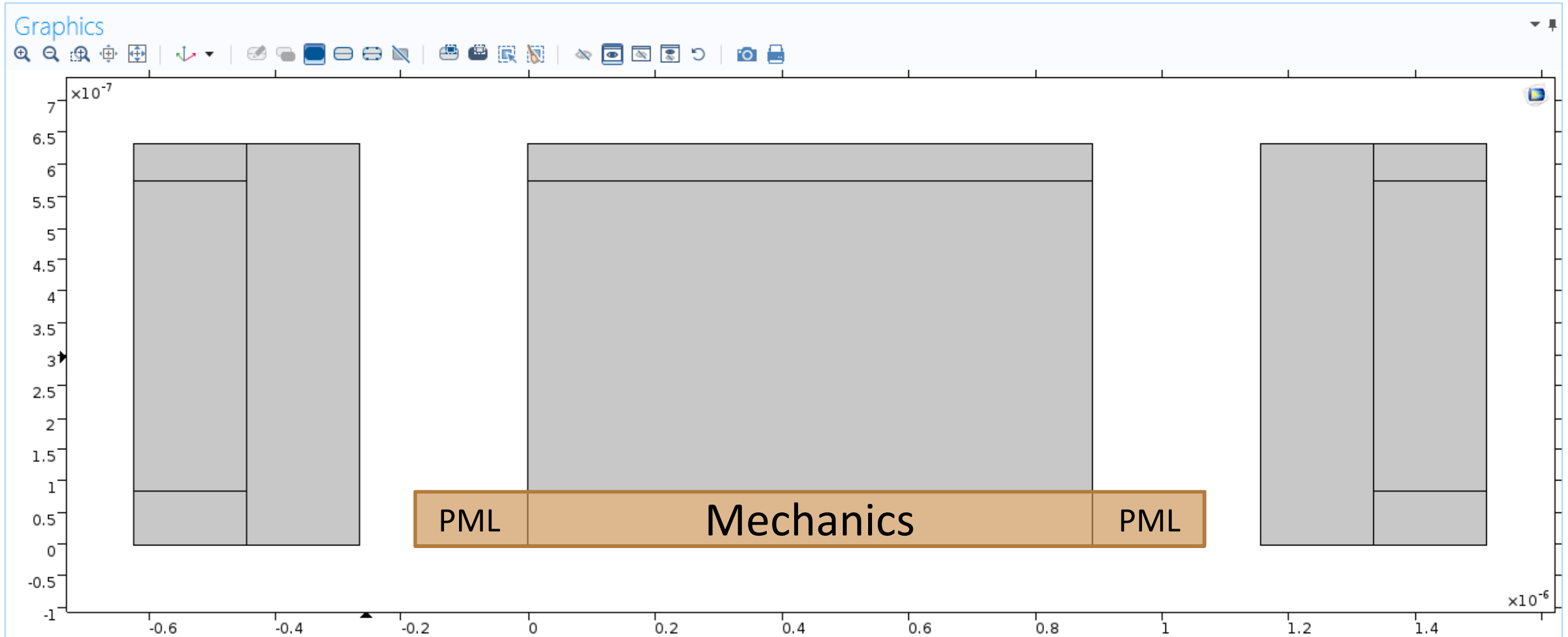


Details of COMSOL simulation: slab model



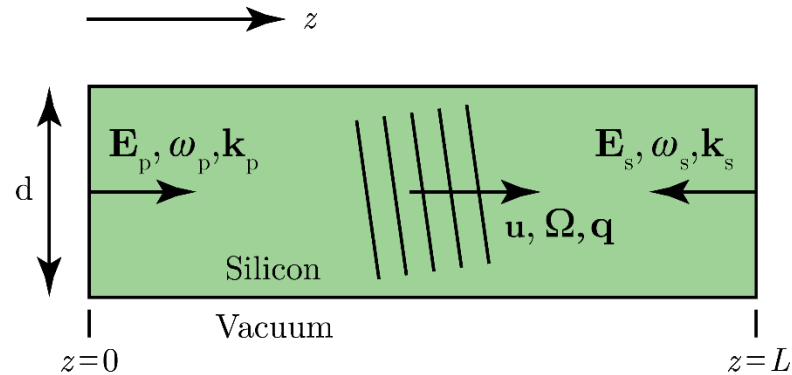


Details of COMSOL simulation: slab model

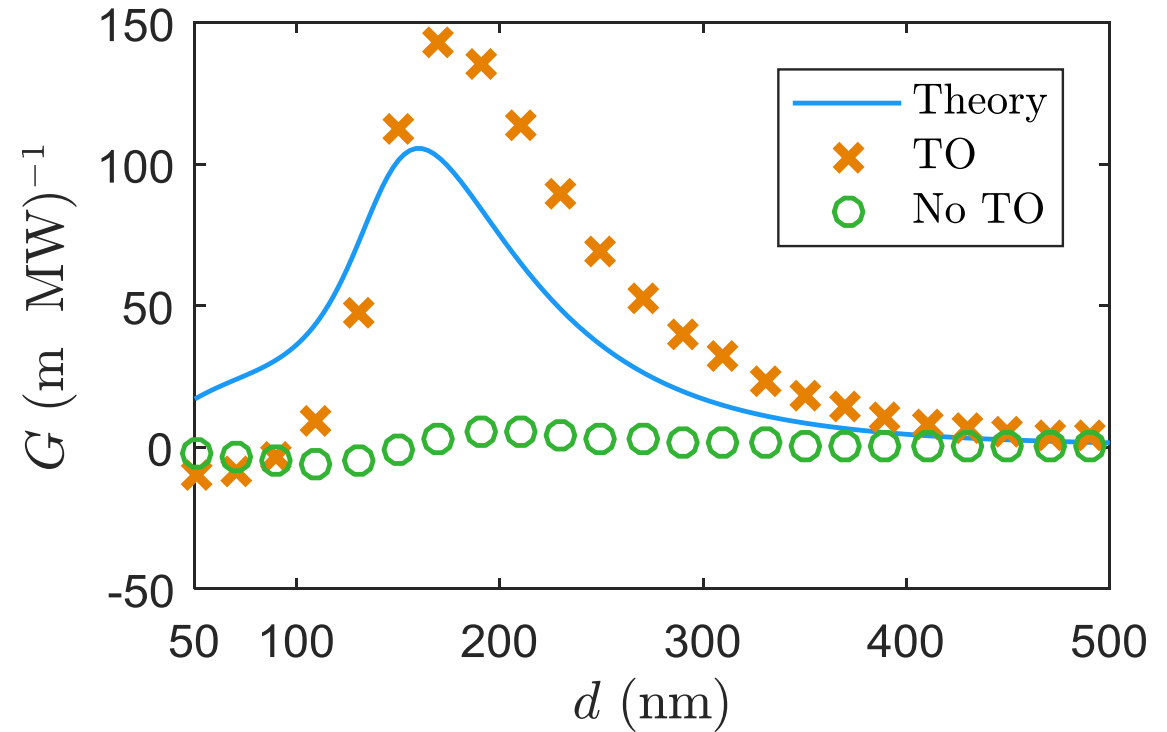




Test: dielectric slab waveguide (details)



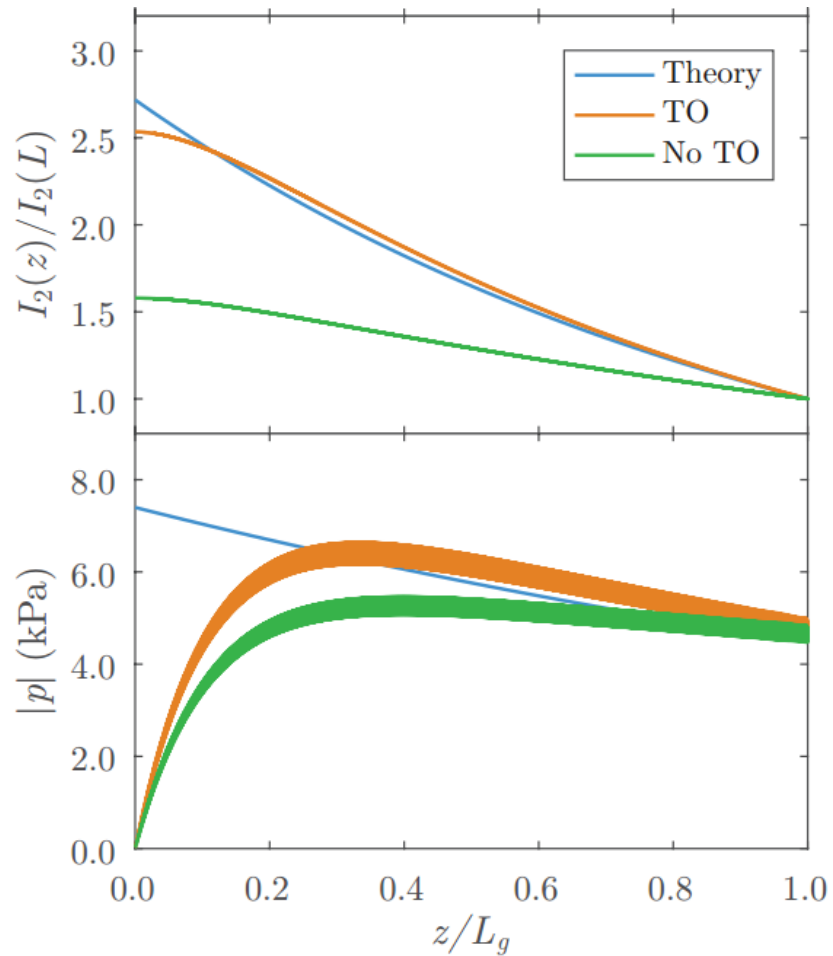
	Pump	Signal	Elastic
Field	\mathbf{E}_p	\mathbf{E}_s	\mathbf{u}
Frequency	ω_p	ω_s	Ω
Wavevector	\mathbf{k}_p	\mathbf{k}_s	\mathbf{q}
Energy cons.		$\omega_p = \omega_s + \Omega$	
Momentum cons.		$\mathbf{k}_p = \mathbf{k}_s + \mathbf{q}$	



Theory calculations based on C. Wolff *et al.*, *Phys. Rev. A* **92** (2015).



Signal fit and gain extraction



$$I_s(z) = I_s(L) e^{gI_p(L-z)}$$

Theory calculations based on R. W. Boyd, *Nonlinear Optics*.



Stress tensors

- Engineering stress tensor

$$\bar{\bar{\zeta}} = \frac{\mathbf{F}}{A_0}$$

- Cauchy stress tensor

$$\bar{\bar{\sigma}} = \frac{\mathbf{F}}{A}$$

- First Piola-Kirchhoff stress tensor

$$\bar{\bar{P}}$$

$$\bar{\bar{S}} = \bar{\bar{F}}^{-1} \bar{\bar{P}}$$

- Second Piola-Kirchhoff stress tensor

$$\bar{\bar{S}}$$

$$\bar{\bar{\sigma}} = \bar{\bar{J}}^{-1} \bar{\bar{P}} \bar{\bar{F}}^T = \bar{\bar{J}}^{-1} \bar{\bar{F}} \bar{\bar{P}} \bar{\bar{F}}^T$$



Definitions of gain

$$\partial_z P_s = G P_p P_s$$

Appropriate for guided systems (\exists cross-section)

$$\partial_z I_s = g I_p I_s$$

Appropriate for bulk (infinite cross-section)