Modal Analysis of Functionally Graded Metal-Ceramic Composite Plates

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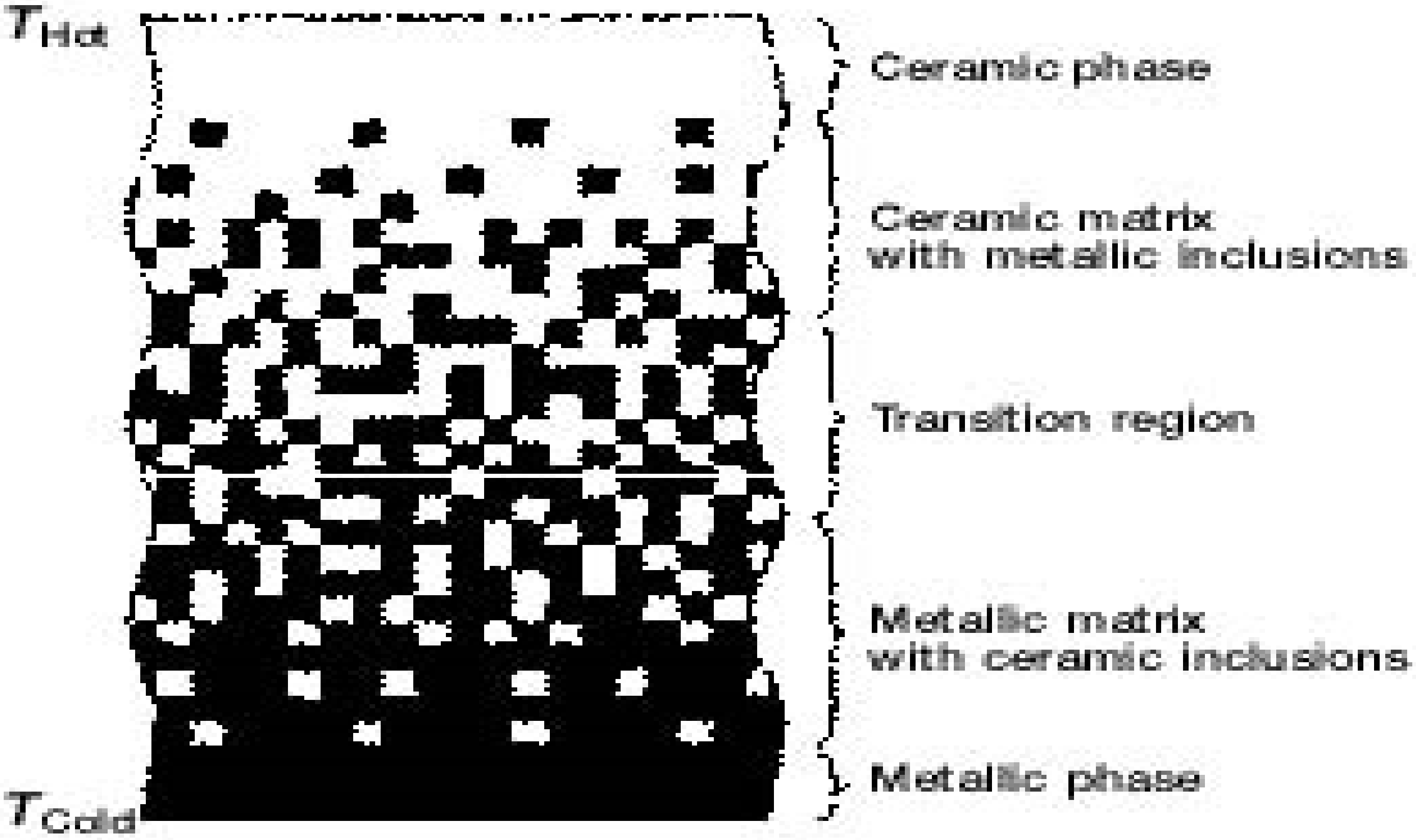
Objectives

- To use the Finite Element Method (FEM) in COMSOL Multiphysics to perform modal analysis of functionally graded materials (FGM) and determine the natural modes of vibration and the mode shapes.
- To compare the results of the COMSOL FEM approximation with other methods of calculation.

Functionally Graded Material



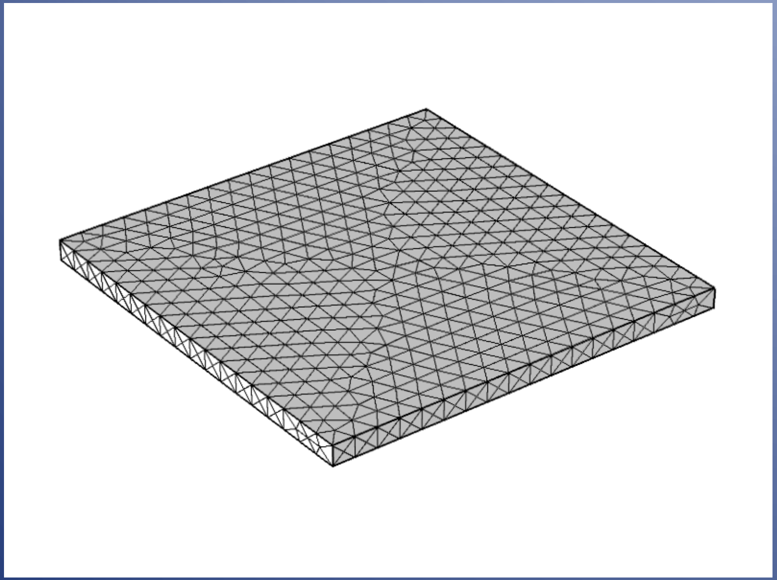
(a) Continuously graded microstructure.

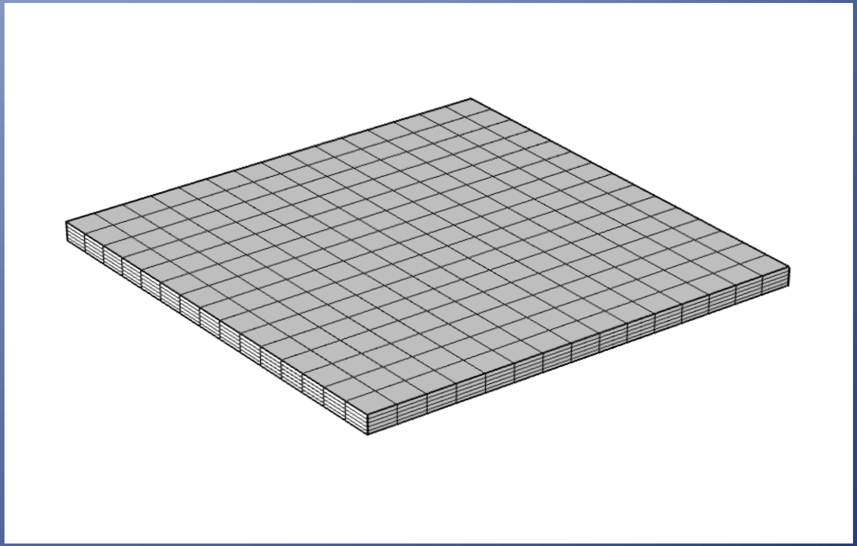
Background

- FGMs
  - FGMs are defined as an anisotropic material whose physical properties vary throughout the volume, either randomly or strategically, to achieve desired characteristics or functionality
  - FGMs differ from traditional composites in that their material properties vary continuously, where the composite changes at each laminate interface.
  - FGMs accomplish this by gradually changing the volume fraction of the materials which make up the FGM.
  - FGMs can be readily produced through 3D Printing
- Modal Analysis
  - Modal analysis involves imposing an excitation into the structure and finding the frequencies at which the structure resonates.

Modal Analysis with Finite Elements

$$(-\omega^2\mathbf{M}+\mathbf{K})\mathbf{u}=0$$





Mori-Tanaka Method

The Mori-Tanaka Method is used to estimate the material properties of the FGM (density ρ, bulk modulus K and shear modulus μ) at any point in the plate as functions of the volume fractions and material properties of the constituent materials

$$\rho_{FGM} = \rho_M V_M + \rho_C V_C$$

$$K_{FGM} = K_M + \frac{(K_C - K_M)V_C}{1 + \frac{(1 - V_C)(K_C - K_M)}{K_M + \left(\frac{4}{3}\right)\mu_M}}$$

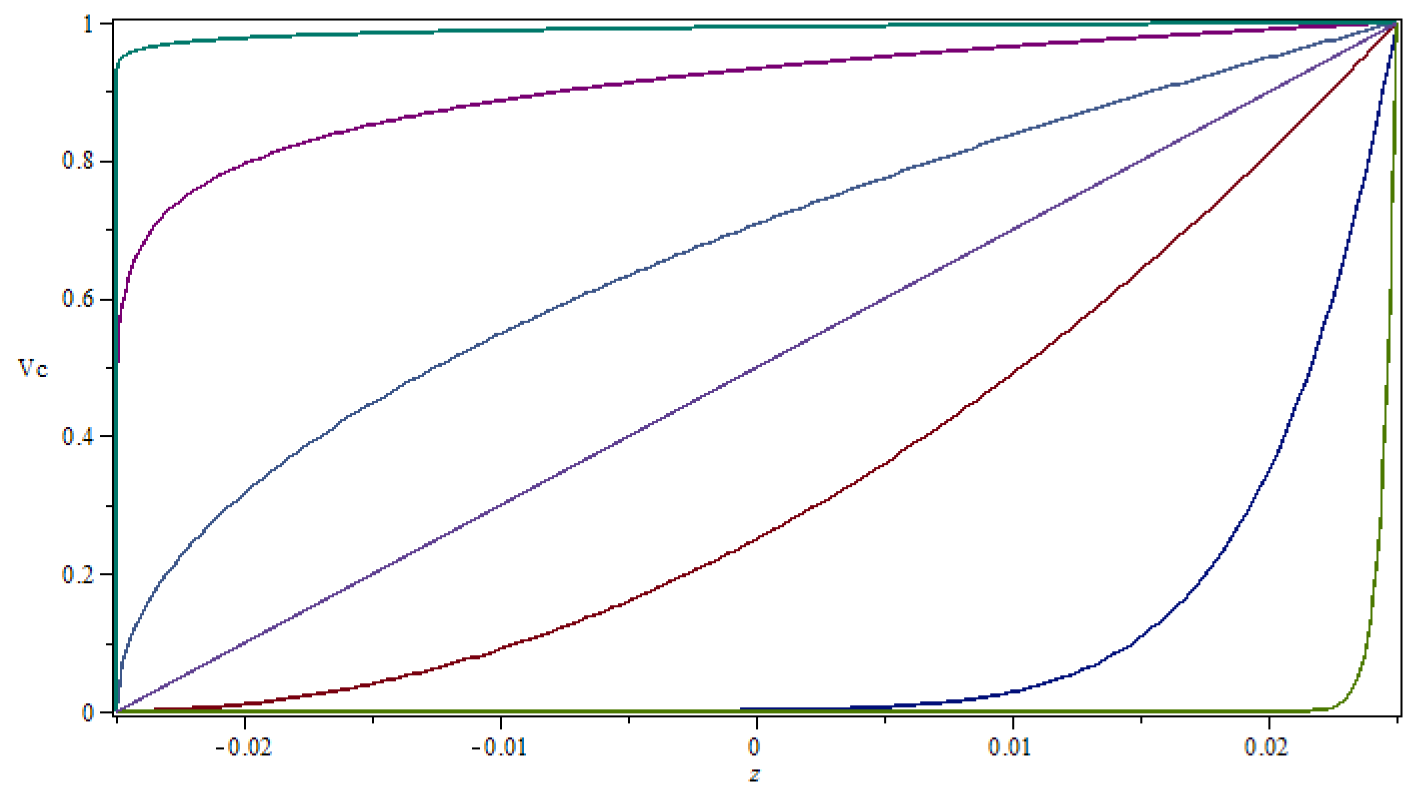
$$\mu_{FGM} = \mu_M + \frac{(\mu_C - \mu_M)V_C}{1 + \frac{(1 - V_C)(\mu_C - \mu_M)}{\mu_M + f_1}}$$

$\lambda = K - (2/3)\ \mu$

$v = [2(1 + \mu/\lambda)]^{-1}$

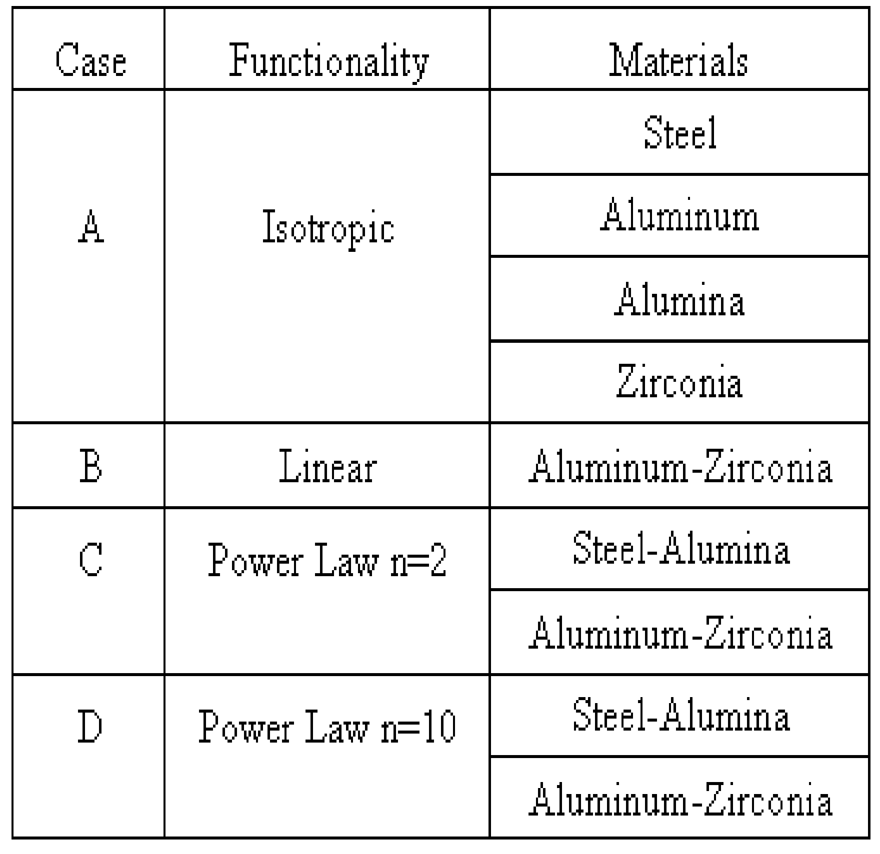
$E = 3\ (1 - 2\ v)\ K$

Volume Fraction of Ceramic through Plate Thickness



Problem Description

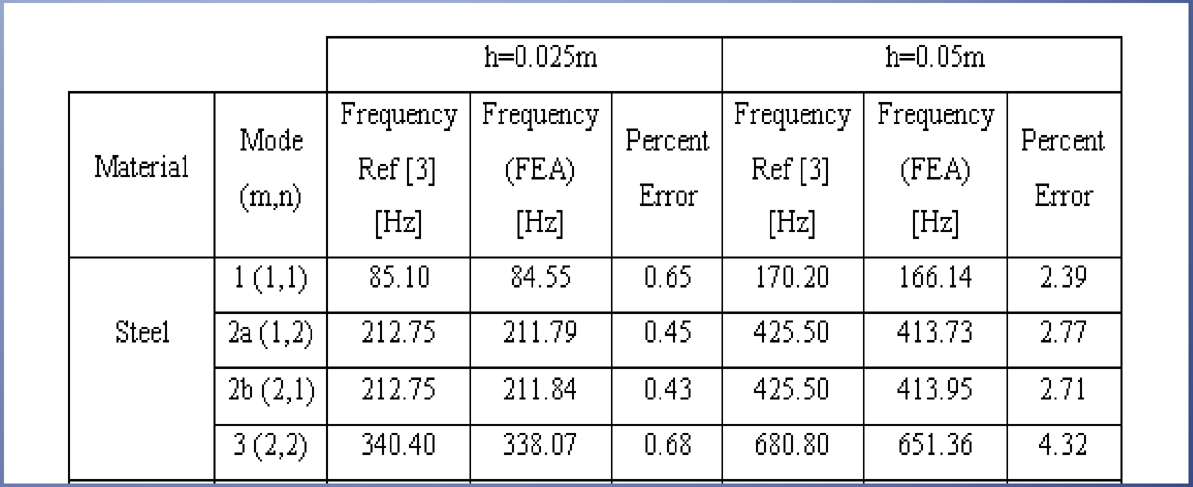
- Each Case
  - Frequencies (4)
  - Mode Shapes (4)
  - Plates 1m x 1m, 0.025m and 0.05m thick
- Case A
  - Compare to theoretical values
- Case B-D
  - Compare to isotropic
- Select Cases
  - Compare to Efraim formula



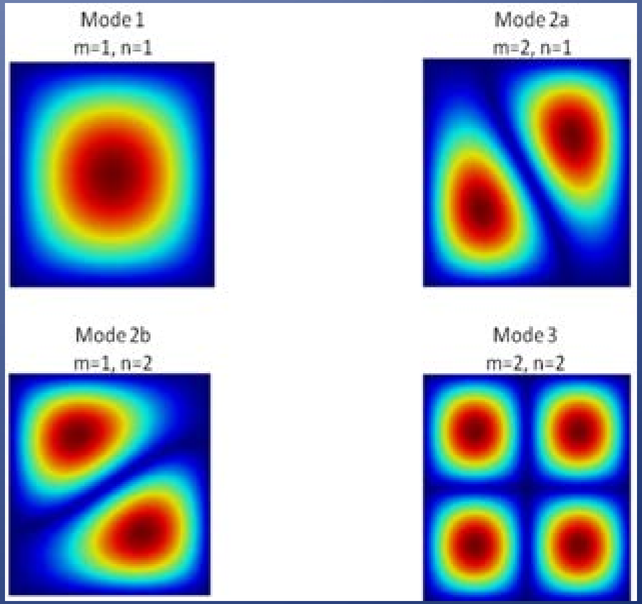
Case	Functionality	Materials
A	Isotropic	Steel
		Aluminum
		Alumina
		Zirconia
B	Linear	Aluminum-Zirconia
C	Power Law n=2	Steel-Alumina
		Aluminum-Zirconia
D	Power Law n=10	Steel-Alumina
		Aluminum-Zirconia

Results – Isotropic Plates

- Isotropic results matched with theory
- Reasons for isotropic case
  - Verify FEA model
  - Check plate thickness limit
  - Have baseline for comparison to FGM

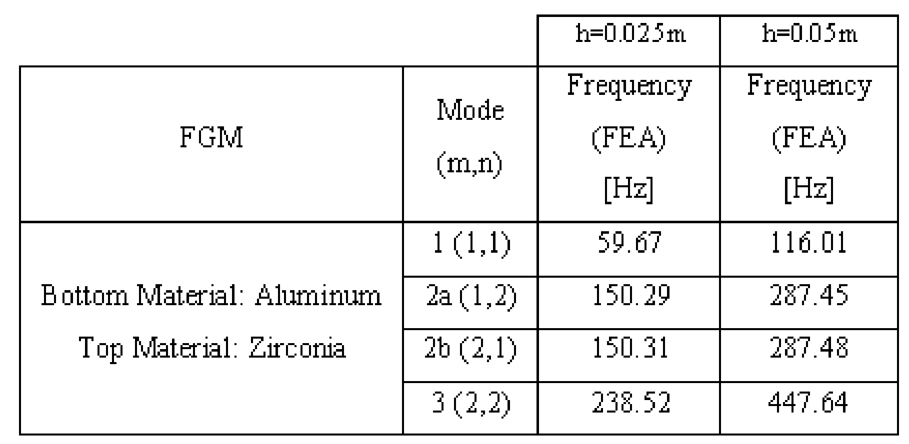


		h=0.025m			h=0.05m		
Material	Mode (m,n)	Frequency Ref [3] [Hz]	Frequency (FEA) [Hz]	Percent Error	Frequency Ref [3] [Hz]	Frequency (FEA) [Hz]	Percent Error
Steel	1 (1,1)	85.10	84.55	0.65	170.20	166.14	2.39
	2a (1,2)	212.75	211.79	0.45	425.50	413.73	2.77
	2b (2,1)	212.75	211.84	0.43	425.50	413.95	2.71
	3 (2,2)	340.40	338.07	0.68	680.80	651.36	4.32

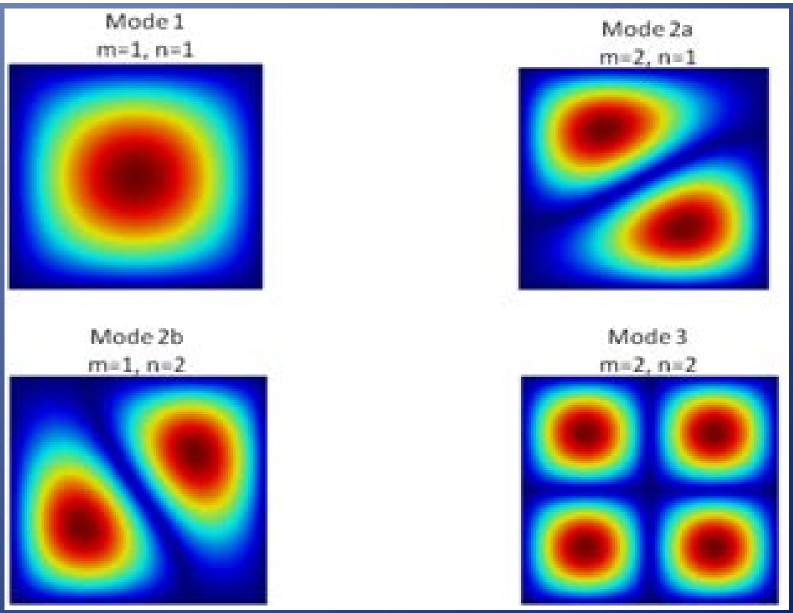


Results – Linear Profile

- Represents, on average, a 50/50 metal-ceramic FGM
- h=0.05m frequencies were bounded by their constituent materials
- Mode shapes 2a and 2b swapped from where they were in isotropic cases

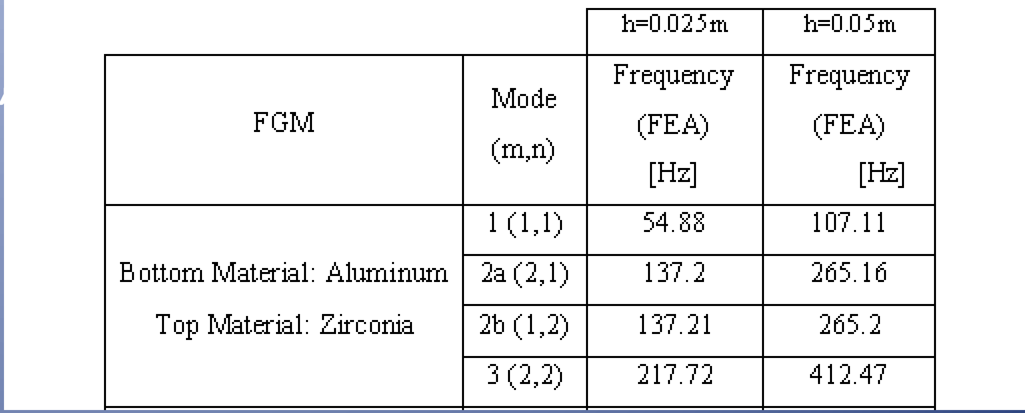


		h=0.025m	h=0.05m
FGM	Mode (m,n)	Frequency (FEA) [Hz]	Frequency (FEA) [Hz]
Bottom Material: Aluminum Top Material: Zirconia	1 (1,1)	59.67	116.01
	2a (1,2)	150.29	287.45
	2b (2,1)	150.31	287.48
	3 (2,2)	238.52	447.64

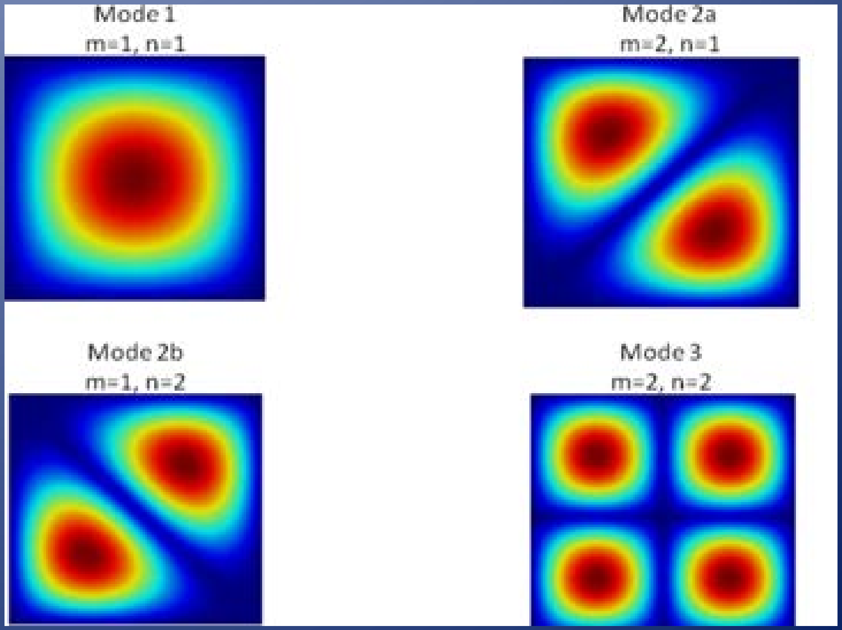


Results – Power Law n=2

- Represents, on average, a 67/33 metal-ceramic FGM
- Frequencies are bounded by their constituent materials
- Mode shapes are changed by addition of ceramic

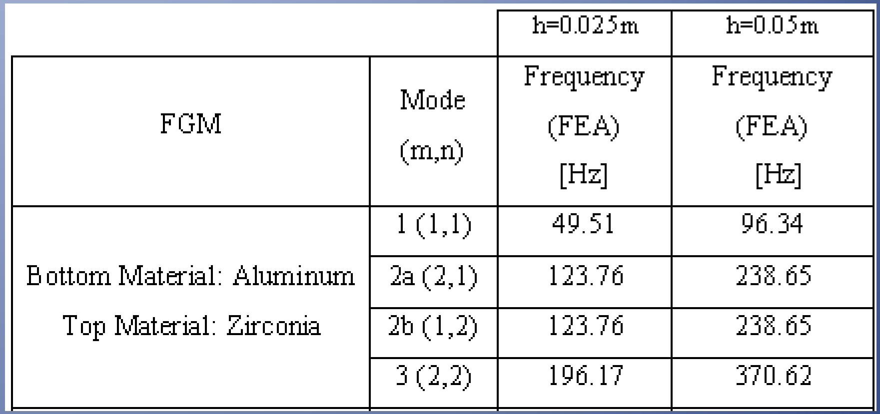


		h=0.025m	h=0.05m
FGM	Mode (m,n)	Frequency (FEA) [Hz]	Frequency (FEA) [Hz]
Bottom Material: Aluminum Top Material: Zirconia	1 (1,1)	54.88	107.11
	2a (2,1)	137.2	265.16
	2b (1,2)	137.21	265.2
	3 (2,2)	217.72	412.47

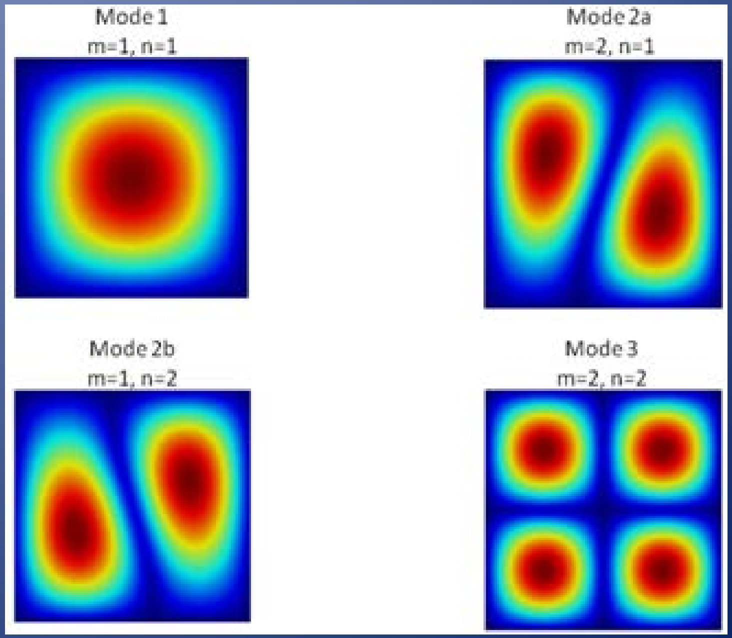


Results – Power Law n=10

- Represents, on average, a 91/9 metal-ceramic FGM, or a metal plate with a thin ceramic coating
- Frequencies were very close to isotropic metal frequencies
- Mode shapes 2a and 2b highly distorted due to presence of ceramic



		h=0.025m	h=0.05m
FGM	Mode (m,n)	Frequency (FEA) [Hz]	Frequency (FEA) [Hz]
Bottom Material: Aluminum Top Material: Zirconia	1 (1,1)	49.51	96.34
	2a (2,1)	123.76	238.65
	2b (1,2)	123.76	238.65
	3 (2,2)	196.17	370.62



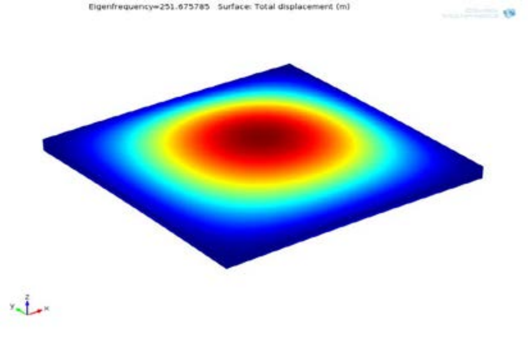
Comparison to Efraim

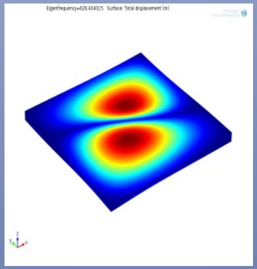
$$f_{FGM} = f_M \sqrt{\frac{\rho_M \cdot E_{eq}}{\rho_{eq} \cdot E_M}} \cdot V_M + f_C \sqrt{\frac{\rho_C \cdot E_{eq}}{\rho_{eq} \cdot E_C}} \cdot V_C$$

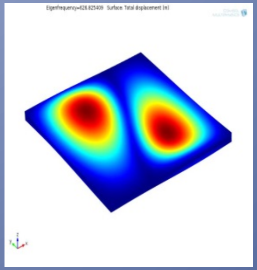
Material	Mode	Frequency ANSYS Hz	Frequency COMSOL Hz	Frequency Efraim ANSYS Hz	Frequency Efraim COMSOL Hz	Error % ANSYS	Error % COMSOL
Steel Alumina p = 10	1	191.06	188.41	188.2	189.46	1.52	-0.55
	2	475.17	467.67	467.51	473.66	1.64	-1.26
	3	475.17	467.67	467.51	473.66	1.64	-1.26
	4	751.29	729.74	738.45	757.86	1.74	-3.71

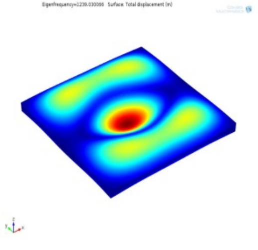
A356-ZrO2

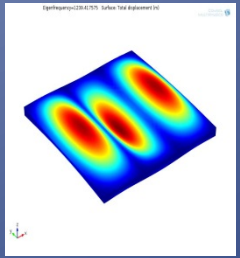
Material	ρ (kg/m^3)	E (Pa)	ν (-)
A356	2670	7.24e9	0.33
ZrO$_2$	5575	1.75e11	0.27

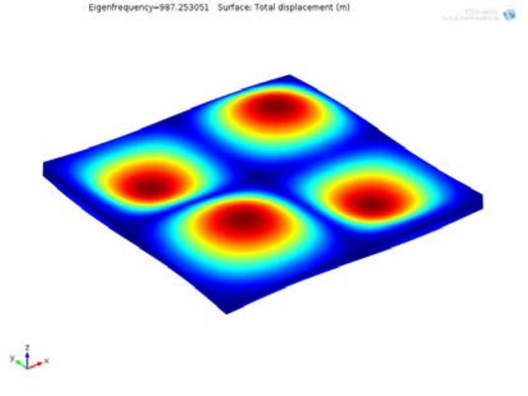


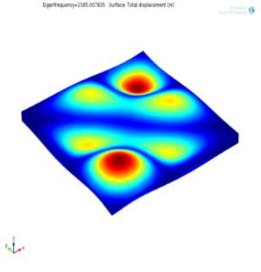


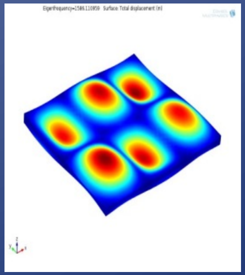












Conclusions

- Modal analysis of FG plates is easily performed using COMSOL Multiphysics
- When considering a FGM that is metal and ceramic, the frequency seems to follow the metal while the mode shape seems to the follow the ceramic
- The FEA results from COMSOL were in good agreement with those computed by Efraim’s formula and also with results obtained using a different FEA program.
- Ongoing work is exploring the application of the methodology for the stress analysis of more complex components produced by 3D printing