Computation of Capacitance, Inductance, and Potential Distribution of Integrated-circuit Interconnects for Unshielded Four conductors with three levels Systems

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Abstract: In this paper an attempt has been made to design and analyze integrated circuit interconnects for unshielded four conductors with three levels systems using Finite Element Method (FEM). The computational and simulation work has been carried out with help of COMSOL Multiphysics software. We illustrate that FEM is as accurate and effective for modeling multilayered multiconductor transmission lines in strongly inhomogeneous media. We mainly focus on designing of two electrostatic models of unshielded four interconnected lines with three levels system. We computed the capacitance and inductance matrices for these configurations. Also, we determine the quasi-static spectral for the potential distribution of the integrated circuits.

Keywords: Finite element method, Capacitance, Inductance, Interconnect, Multiconductor

1. Introduction

Computational methods of electromagnetic used in analyzing and designing of electronic interconnects and packages become an important area in industry due to the rapid increase in operating frequencies and feature size of integrated circuits systems. Many contributions have been discussed the computational methods for analyzing the complicated integrated circuits with applicable computer storage and executing time.

There are several computational methods used at the problem include using the measured equation of invariance (MEI) method [1-2], on-surface measured equation of invariance (MEI) method [3-5], and the method of moments (MoM) [6-8].

In this work, we design of two electrostatic models of unshielded four interconnected lines with three levels system using the finite element method (FEM) with COMSOL multiphysics package. Many industrial applications depend on different interrelated properties or natural phenomena and require multiphysics modeling and simulation as an efficient method to solve their engineering problems. Moreover, superior simulations of microwave integrated circuit applications will lead to more cost-efficiency throughout the development process. We specifically calculate capacitance and inductance per unit length matrices, and the potential distribution of the configurations.

2. Results and Discussions

The models are designed in 2D using electrostatic environment in order to compare our results with some of the other available methods. In the boundary condition of the model's design, we use ground boundary which is zero potential $(V = 0)$ for the shield. We use port condition for the conductors to force the potential or current to one or zero depending on the setting. Also, we use continuity boundary condition between the conductors and between the conductors and left and right grounds.

The quasi-static models are computed in form of electromagnetic simulations using partial differential equations. In this paper, we consider two different models. Case A investigates the designing of unshielded four interconnected lines with three levels system, two conductors in the substrate at different levels. In Case B, we illustrate the modeling unshielded four interconnected lines with three levels system, two conductors in the substrate are situated on top of each other.

For the models, the capacitance matrix entries are computed from the charges that result on each conductor when an electric potential is applied to one of them and all the others are set to ground. The matrix is defined as follows:

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \cdot \\ \cdot \\ \cdot \\ Q_N \end{bmatrix} = \begin{bmatrix} C_{11}V_1 + C_{12}V_2 + \ldots + C_{1N}V_N \\ C_{21}V_1 + C_{22}V_2 + \ldots + C_{2N}V_N \\ \cdot \\ \cdot \\ \cdot \\ C_{N1}V_1 + C_{N2}V_2 + \ldots + C_{NN}V_N \end{bmatrix} \qquad (1)$$

where Q_N is the charge per unit length on a conductor N, V_N is the potential of a conductor N, with respect to the ground (conductor $N+1$), C_{NN} is the capacitance per unit length between the N th conductor and the ground plane, C_{Nj} is the coupling capacitance per unit length between the N th conductor and j th.For example, using port 2 as the input will provide the entries of the second column: C_{12}, C_{22}, ..., C_{N2}.

Now, the inductance and capacitance of multiconductor transmission lines are related as

$$[L] = \mu_o \varepsilon_o [C_o]^{-1} \qquad (2)$$

where

$[L]$ = inductance matrix,

$[C_o]^{-1}$ = the inverse matrix of the capacitance of the multiconductor transmission line when all dielectric constants are equal to 1,

ε_0 = permittivity of free space or vacuum,

μ_0 = permeability of free space or vacuum.

The models designed with finite elements are unbounded (or unshielded), meaning that the electromagnetic fields should extend towards infinity. This is not possible because it would require a very large mesh. The easiest approach is just to extend the simulation domain “far enough” that the influence of the terminating boundary conditions at the far end becomes negligible. In any electromagnetic field analysis, the placement of far-field boundary is an important concern, especially when dealing with the finite element analysis of structures which are open. It is necessary to take into account the natural boundary of a line at infinity and the presence of remote objects and their potential influence on the field shape [9]. In our simulations for the unshielded models, we surrounded them by a $W \times H$ shield, where W is the width and H is the thickness.

2.1 Modeling of Unshielded Four Interconnected Lines with Three Levels System, Two Conductors in the Substrate at Different Levels

Figure 1 shows geometry of the model with following parameters:

ε_{r1} = dielectric constant = 5.0;

ε_{r2} = dielectric constant = 1.0;

w = width of the conductors = 1mm;

s =distance between conductors 1 and 2 = 1mm;

t = thickness of the conductors = 0.01mm;

H_1 = height of dielectric layer 1 from the ground = 3mm;

H_2 = height of conductor 4 from the ground = 1mm;

H_3 = height of conductor 3 from the ground = 2mm;

The geometry is enclosed by a 18×15 mm shield.

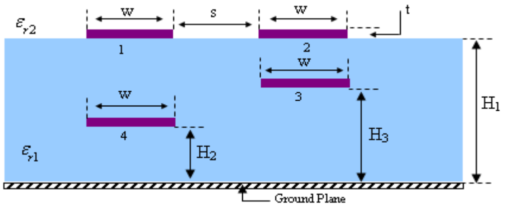

Figure 1. Cross section of unshielded four interconnected lines with three levels system, two conductors in the substrate at different levels.

Figure 2 shows the three-dimensional (3D) surface potential distribution of the transmission lines, while, the mesh plot was presented in Figures 3. Table 1 shows the statistical properties of the mesh.

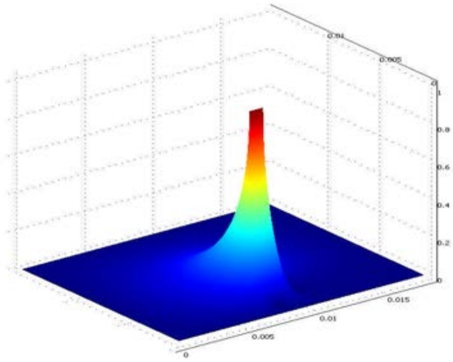

Figure 2. 3D surface potential distribution of unshielded four interconnected lines with three levels system, two conductors in the substrate at different levels.

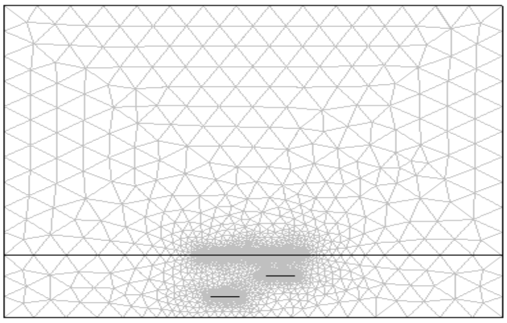

Figure 3. Mesh plot of unshielded four interconnected lines with three levels system, two conductors in the substrate at different levels.

Table 1 Mesh statistics of the model

Items	Value
Number of degrees of freedom	28862
Total Number of mesh points	6907
Total Number of elements	13743
Triangular elements	13743
Quadrilateral	0
Boundary elements	789
Vertex elements	22

Contour plot of this unshielded model presented in Fig. 4. In addition, Fig. 5 presents the electric potential plot as a function of arc-length. Fig. 6 shows the comparison analysis of potential distribution of the model with and without dielectric substrate. It observed that the peak value of electric potential is decreased as the dielectric is placed in the substrate.

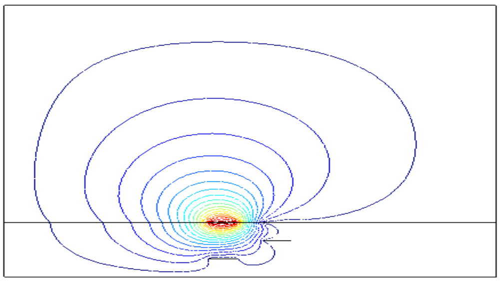

Figure 4. Contour plot of unshielded four interconnected lines with three levels system, two conductors in the substrate at different levels.

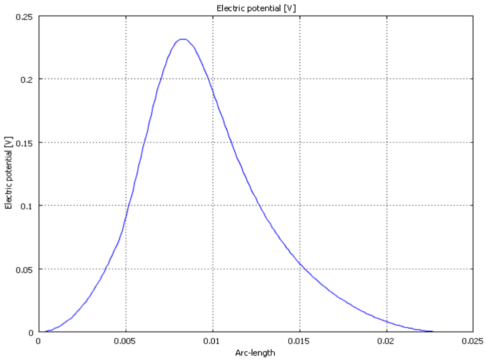

Figure 5. Potential distribution of unshielded four interconnected lines with three levels system, two conductors in the substrate at different levels from (x,y) = (0,0) to (x,y) = (18,15) mm.

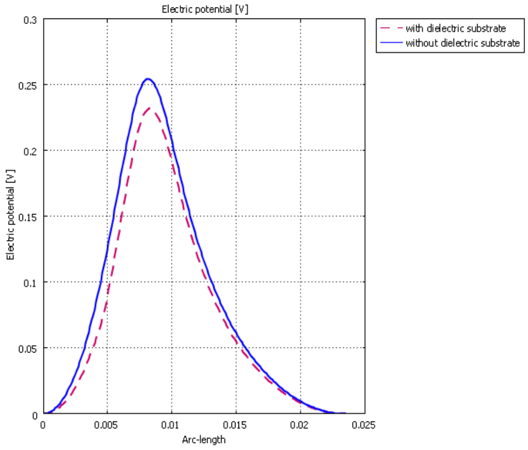

Figure 6. Comparison analysis of potential distribution of the model with and without dielectric substrate.

Table 2 shows the COMSOL results for the capacitance per unit length of the model compared with the work of previous investigators using MoM and OSMEI methods. They are in good agreement.

Table 2 Capacitance matrix [C] of the model in Fig. 1.

Capacitance (pF/m)	MoM	OSMEI	Our work
C11	70.158	69.514	72.103
C12	-12.842	-12.832	-13.232
C13	-12.960	-13.110	13.405
C14	-22.240	-23.014	-23.088
C22	87.327	87.028	90.221
C23	-54.195	-55.462	-56.566
C24	-4.052	-3.988	-4.022
C33	133.935	128.861	138.656
C34	-15.606	-14.935	-16.592
C44	141.170	141.312	145.560

The inductance per unit length matrix ($[L]$), using equation (2), for the model is:

$$[L]=\begin{bmatrix} 0.6016 & 0.2181 & 0.1665 & 0.1238 \\ 0.2181 & 0.6029 & 0.2835 & 0.0861 \\ 0.1666 & 0.2836 & 0.5362 & 0.0951 \\ 0.1238 & 0.0861 & 0.0951 & 0.4124 \end{bmatrix} \mu F/m^{\cdot}$$

2.2 Modeling of Unshielded Four Interconnected Lines with Three Levels System, Two Conductors in the Substrate are Situated on Top of Each Other

In this section, we illustrate the modeling of new unshielded four interconnected lines with three levels system; two conductors in the substrate are situated on top of each other. We focus on the calculation of capacitance and inductance per unit length matrices and determine the quasi-TEM spectral for the potential distribution of the model. Figure 7 shows geometry of the model with similar parameters values as in the previous model.

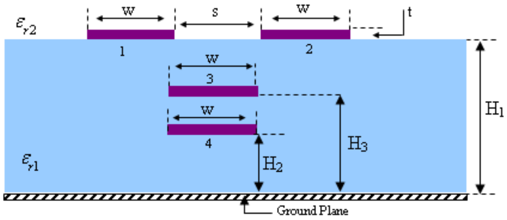

Figure 7. Cross section of unshielded four interconnected lines with three levels system, two conductors in the substrate are situated on top of each other.

Figure 8 shows the 2D surface potential distribution of the transmission lines, while, the mesh plot was presented in Figures 9. Table 3 shows the statistcal properties of the mesh for this model.

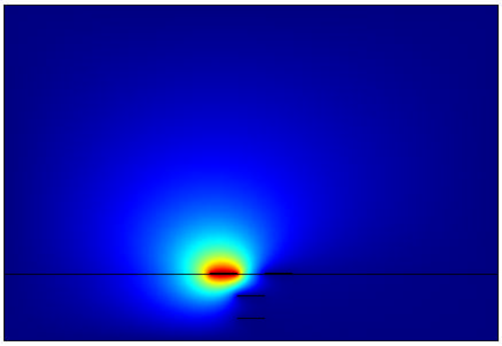

Figure 8. 2D surface potential distributions of unshielded four interconnected lines with three levels system, two conductors in the substrate are situated on top of each other.

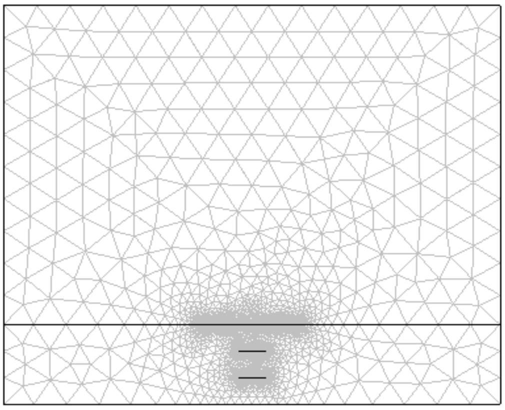

Figure 9. Mesh of unshielded four interconnected lines with three levels system, two conductors in the substrate are situated on top of each other.

Table 3 shows the statistical properties of the mesh for the model.

Table 3 Mesh statistics of the model

Items	Value
Number of degrees of freedom	28313
Total Number of mesh points	6770
Total Number of elements	13470
Triangular elements	13470
Quadrilateral	0
Boundary elements	787
Vertex elements	22

However, Contour plot of this unshielded model presented in Fig. 10. Fig. 11 shows the comparison analysis of potential distribution of the model with and without dielectric substrate. It observed that the peak value of electric potential is decreased as the dielectric is placed in the substrate

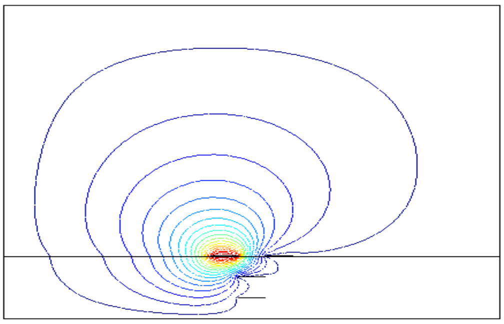

Figure 10. Contour plot of the model.

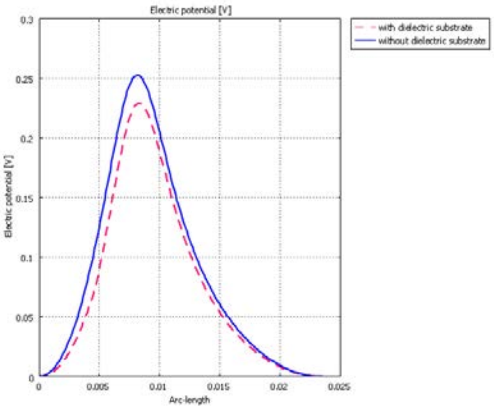

Figure 11. Comparison analysis of potential distribution of the model with and without dielectric substrate.

Table 4 shows the COMSOL results for the capacitance per unit length of the model.

Table 4 Capacitance matrix [C] of the model in Fig. 7.

Capacitance (pF/m)	Our work
C11	78.879
C12	-10.834
C13	-36.387
C14	-6.477
C22	78.879
C23	-36.387
C24	-6.477
C33	154.359
C34	-54.192
C44	163.347

The inductance per unit length matrix ($\left[L\right]$), using equation (2), for this model is:

$$[L]=\begin{bmatrix} 0.5991 & 0.2222 & 0.2369 & 0.1125 \\ 0.2222 & 0.5990 & 0.2368 & 0.1124 \\ 0.2369 & 0.2368 & 0.5320 & 0.1949 \\ 0.1125 & 0.1125 & 0.1949 & 0.4133 \end{bmatrix} \mu F/m$$

3. Conclusions

In this paper we have presented the capacitance and inductance matrices of unshielded four interconnected lines with three levels systems in 2-D case, have been extracted using FEM with COMSOL multiphysics. Also, we determine the quasi-TEM spectral for the potential distribution of the multiconductor transmission lines in multilayer dielectric media. The results obtained in this research are encouraging and motivating for further study.

4. References

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