Xylophone Bar Magnetometry and Inertial Grade MEMS Optimisation

A Multiphysics Approach

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Who are we?
What do we do?
What is this research about?

- MEMS and microsystems research group with world-leading expertise in dynamics and control, biosensors
- Based in the North East of England
- A major aim is to demonstrate the feasibility of inertial grade navigation , similar to aviation-grade ILS, using an integrated microsensor array – the 9 DOF "holy grail"

- Consistent publications and current EPSRC sponsored work on high-Q resonant sensors and associated parametric control techniques
- See eg. Gallacher et al., *Sens.Act.A*, 2010 for work on high-precision MEMS ring gyros

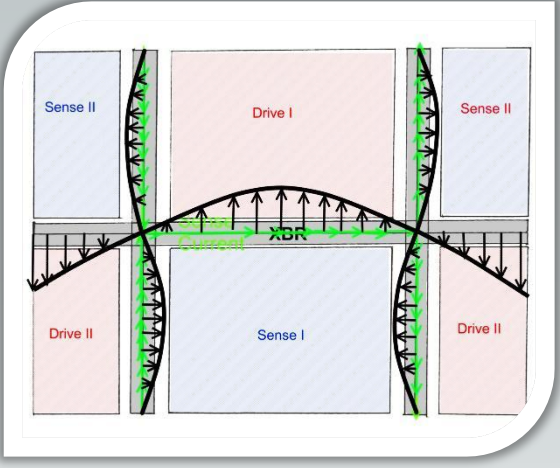
- The work presented here today is a subset of my doctoral research
- My work is focused on developing a resonant Lorenz micromagnetometer
- In an inertial navigation context, would be employed in a Kalmann filter to implement drift nulling

What is a Xylophone Bar Magnetometer? Why should I care?

- An XBM is a resonant high sensitivity Lorenz magnetometer

- When a DC sense current is applied, any B field component transverse to the plane of vibration generates a Lorenz force and corresponding deflection of the structure
- When the sense current is made to oscillate at the resonant frequency of a chosen mode of the XBR structure, the static deflection is amplified by the Q factor of the mode

- Differs from other Lorenz magnetometers in that the suspension beams are attached at the node points of the sense beam, decoupling transverse motion between the two
- This gives extremely high Q factors and hence sensitivities
- Q factor can be pushed higher using parametric drive techniques

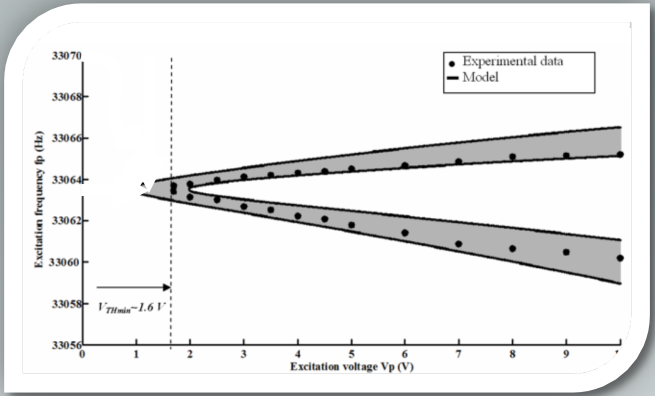


XBR Operation

XBM control relation is known

- Quantitatively expresses dependence of device sensitivity on Q, static compliance
- Experimental work gives the achievable limiting parametric gain at ~100
- Implies a classical resonator Q of 10^4 would yield an effective parametric Q on order 10^6 without affecting compliance

$$A_0 = \frac{iF\omega}{2}[Q \times G_T]$$



It implies that the Q factor is critical

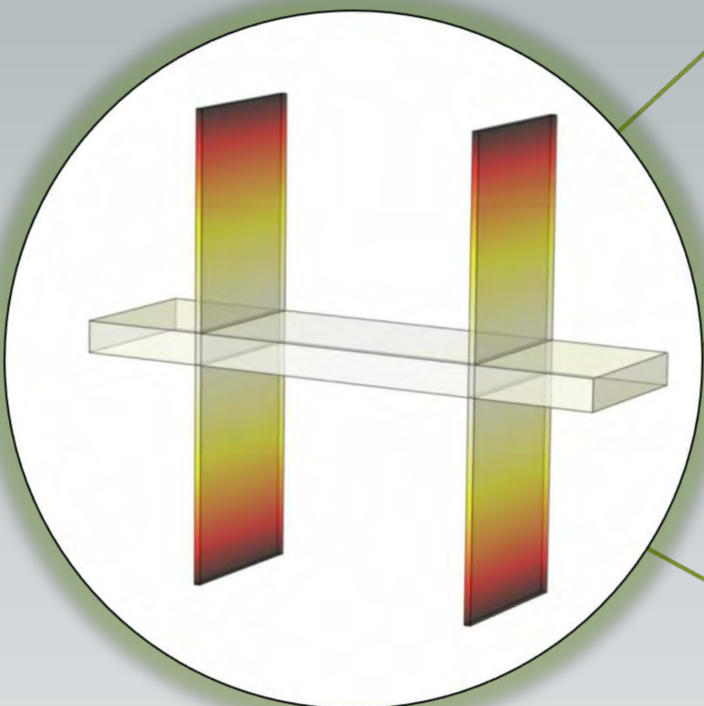
- Need an understanding of Q factor and dissipation mechanisms to optimise design
- Gas damping, surface losses, TED, etc. well charaterised in the literature.
- TED sets a hard limit on Q at ~ 10^6
- No such results exist for support loss at present in the literature

Strategy: Develop and validate a model of support loss in XBRs

- Model the support losses in COMSOL.
- Use a Rayleigh-Ritz method to obtain the forces of constraint at the distal ends of the XBR support beams
- Use 2D analytical model of elastic wave radiation in a semi-infinite plane to estimate corresponding support radiation
- Cross-validate simulation and analytical results

How can we maximise the performance of an XBM?

Model I: Joule Heating



$$\nabla \cdot \boldsymbol{J} = Q_j \; ; \; \boldsymbol{J} = \sigma \boldsymbol{E} + \boldsymbol{J}_{\boldsymbol{E}}$$

$$E = -\nabla \boldsymbol{V} \; ; \; \rho C_p \boldsymbol{u} \nabla T = \nabla \cdot (k \nabla T) + Q$$

XBR uses an AC sense current to generate Lorenz force

- Scaled with field strength to give wide dynamic range
- The larger the current, the smaller a field can be detected

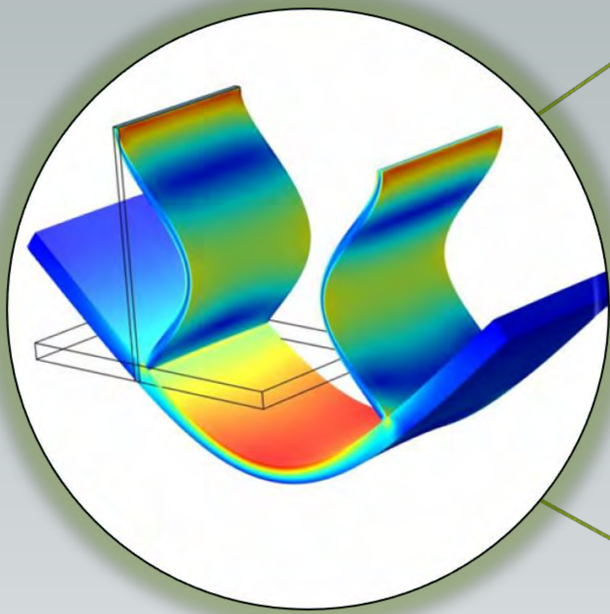
The sense current amplitude is limited by Joule Heating of the resonator

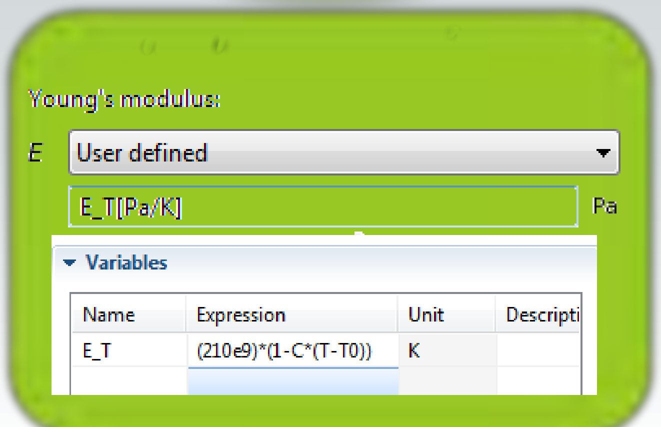
- Use Joule Heating model, Stationary study type, to find steady state temperature distribution

Under vacuum, only radiation and conduction important

- Surface to surface radiation ignored
- Thermal BCs specify radiation to ambient environment and a prescribed temperature on the distal ends of the supports
- Electrical BCs specify a potential at the distal ends of the supports parametrically, with the symmetry boundary taken as a ground

Model 2: Structural Mechanics





Variable density, Young's modulus

- An analytical expression for the temperature dependence of ρ and E is given in _ref_
- Implemented as a variable in COMSOL and evaluated pointwise over the computational domain
- Used to define the material properties for Model 2

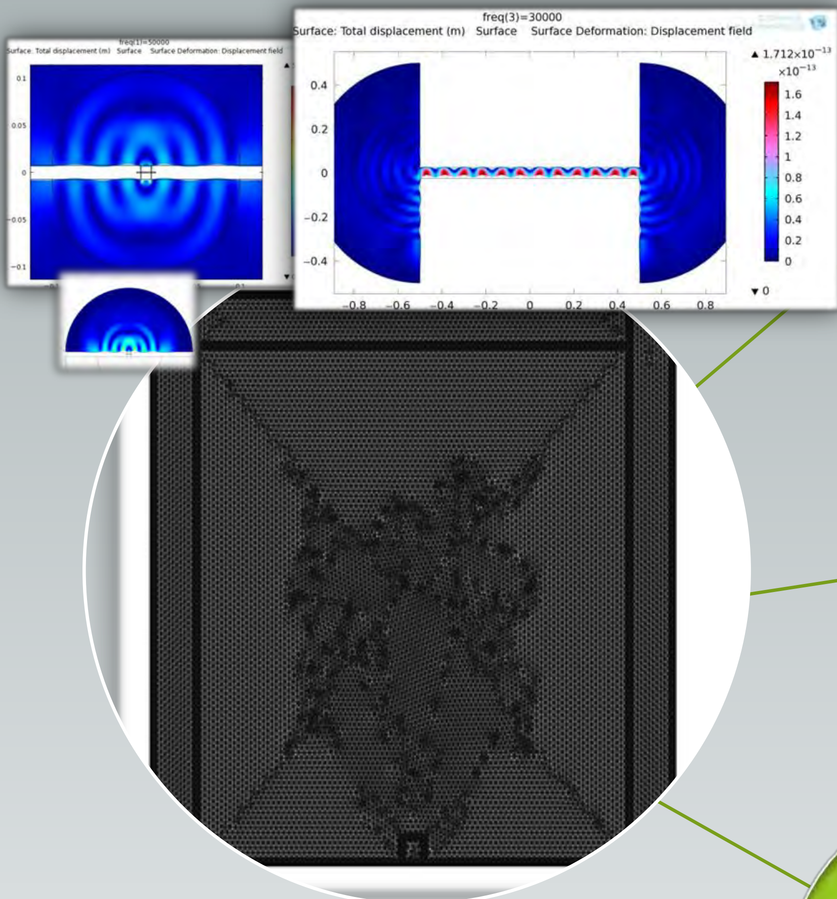
Eigenvalue study of Solid Mechanics model

- At ambience, support and sense beams mode-matched
- This minimises constraint forces and hence optimises support Q
- Sensitivity to geometric mistuning studied using COMSOL in Grigg et al., IMAPS DPC, 2011.

Heating-induced mistuning

- Domain rendered inhomogeneous by spatial dependence of temperature
- Wave propagation and consequentially normal modes and their frequencies altered
- This problem is analytically challenging, but made straightforward by COMSOL.

Model III: PML



Perfectly Matched Layers are a numerical technique used to simulate infinite domains

- Best viewed as an analytical continuation of the constitutive equations to the complex plane

Simulates energy loss to the substrate

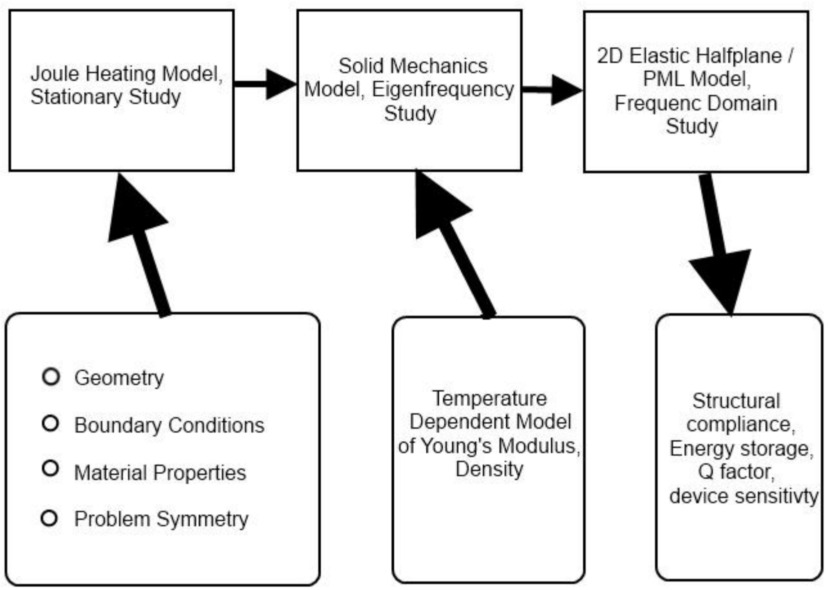
- Mechanism of support loss is elastic wave radiation
- PML allows numerical closure and hence simulation

Model solved using Frequency Domain study

- Resonant frequency of the XBR and force distribution at the support interfaces used as model inputs
- Separate geometric model employed

$$\frac{\partial}{\partial x} \rightarrow \frac{1}{1 + i\sigma(x)/\omega} \frac{\partial}{\partial x}$$

Multiphysics Handling and Solver Flow



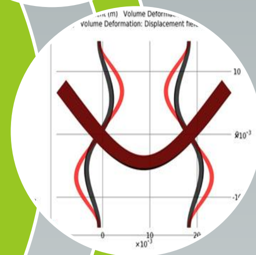
F3: Job sequence for coupled multiphysics analysis of XBR support loss. This sequence was iterated in a 2-parameter parametric array.

- Geometry modelled parametrically in COMSOL
- The steady-state temperature distribution deriving from an applied sense current and Joule heating studied first
  - The above output was combined with an experimental result from the literature was used to define a variable Young's modulus and density for the resonator material
- Mode shapes and natural frequencies found using a linear elastic eigenfrequency analysis with material properties defined by the previous result
  - Constraint forces found -> PML model input
  - In addition, the stored strain energy in the resonator was determined using a volume integration probe **(Result 1)**.
- The constraint forces were then coupled to a PML model approximating the resonator substrate as large (and hence entirely dissipative).
  - The total energy flux arising from elastic wave propagation into the substrate could thus be determined **(Result 2)**.
- Combining results **1** and **2** yields an estimate for the device Q factor, as desired.

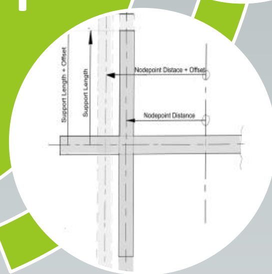
Results

Parametric Geometric Optimisation

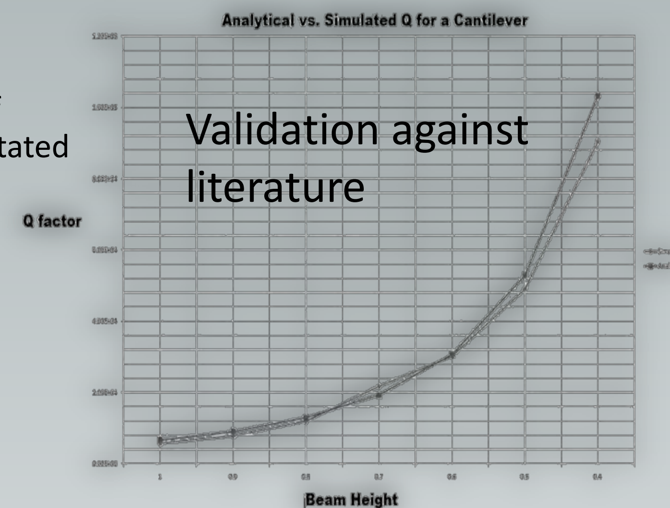
Temperature-modified mode shapes found



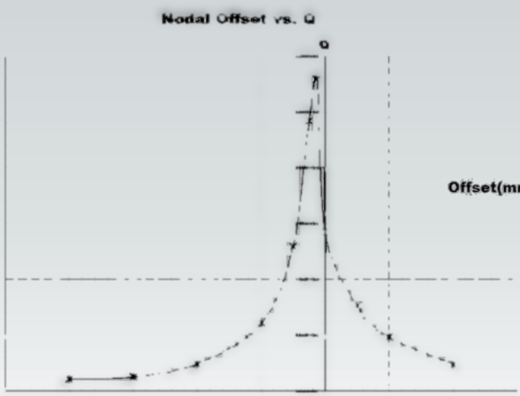
Parametric optimisation of geometry facilitated

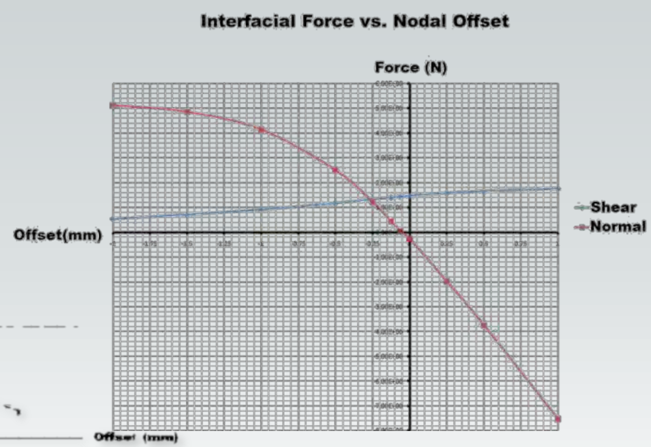


Optimal tuning is a function of applied sense current – retuned for small-field sensitivity



Validation against literature



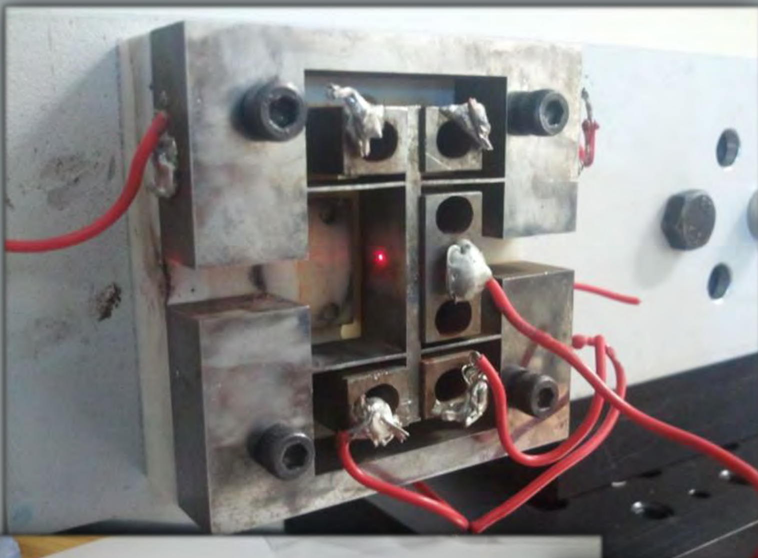


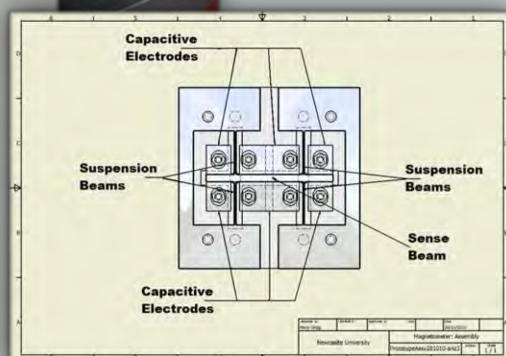
Q vs Geometry

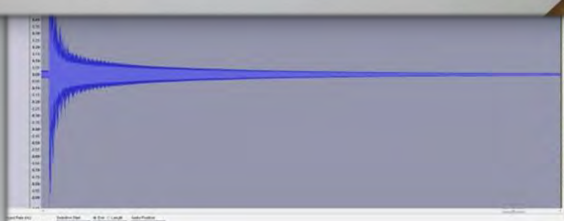
Nodal Mistuning(%)	-0.5	0	0.5
V0=0	26	41	29
V0=0.02	28	39	28
V0=0.04	32	31	25

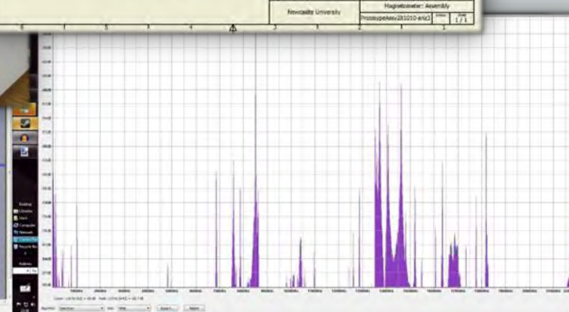
Q ('000) vs Parameters

Prototypes and Progress









- ❑ Two prototypes to date
- ❑ Characterised optically and acousticaly
- ❑ Q around 10 000 at 1 bar!!
- ❑ Results of the present study suggest thermal limitation of performance
- ❑ As a result, a third prototype is under production in copper – superior heat dissipation

Conclusions

- Support loss, natural frequencies, and static compliance efficiently modelled using COMSOL
  - Analytical work cross-validated against the results with satisfactory agreement for the case without heating effects
  - Model predictions extended using coupled analysis to include heating
- Leads to adjustment of the optimal geometric tuning for an XBR
  - Improved performance in the real world

- **Q factor under vacuum implies inertial-grade performance possible**

Further Work

- Complete prototyping, obtain experimental validation of modelling
  => Proof of concept
- Obtain funding, make the device on the microscale
- Analytically model heating effects – finite difference method?
- Model Parametric Drive

- Realise the magnetic component of 9-DOF IMU

- Many thanks for listening to my talk!
- My enduring gratitude to Barry Gallacher and NU Microsystems.
- This work would not have been possible without financial support from the EPSRC.
- Finally, thanks to COMSOL for t awesome software and the opportunity to present my work

Thanks for your attention!

Any Questions?