Pull-In Voltage for a Biased Resonator—3D

Introduction

Silicon micromechanical resonators have long been used for designing sensors and are now becoming increasingly important as oscillators in the consumer electronics market. In this sequence of models, a surface micromachined MEMS resonator, designed as part of a micromechanical filter, is analyzed in detail. The resonator is based on that developed in Ref. 1.

This model performs a pull-in analysis of the structure, to predict the point at which the biased system becomes unstable. The analysis begins from the stationary analysis performed in the accompanying model Stationary Analysis of a Biased Resonator— 3D; please review this model first.

Model Definition

The geometry, fabrication, and operation of the device are discussed for the "Stationary Analysis of a Biased Resonator—3D" model.

This model computes the pull-in voltage for the resonator by solving an inverse problem. The z-coordinate of the resonator midpoint is computed using an integration operator (`intop1`). The inverse problem that COMSOL solves computes the DC voltage that must be applied to the beam in order to move the midpoint to a set z-coordinate, `zset`. This is achieved by adding a global equation for the DC voltage, `VdcSP`, applied to the resonator. The equation `intop1(z)-zset=0` is solved to determine the value of `VdcSP`. This means that `VdcSP` is adjusted until the midpoint of the resonator has a z-coordinate given by the set value, `zset`. Essentially COMSOL is being asked to find the voltage that allows the beam to exist in equilibrium (stable or unstable) at a given displacement. Solving the problem in this way avoids complications with trying to solve a problem with no solution (which is what happens if the voltage is continuously ramped up eventually exceeding the pull-in voltage). The result of the analysis is a displacement versus voltage plot, with a minimum at the pull-in voltage. Note that for a linear spring, the pull-in displacement corresponds to 1/3 of the gap distance. Although the inclusion of geometric nonlinearities in the solid

mechanics solver means that the pull-in displacement changes slightly from this value, it is usually most efficient to search around this point for the pull-in voltage.

Results and Discussion

Figure 1 shows the voltage-displacement curve for the resonator at equilibrium. The y-coordinate at which the pull in occurs corresponds to a displacements around 1/3 of the gap size. The pull-in voltage is 59.1 V.
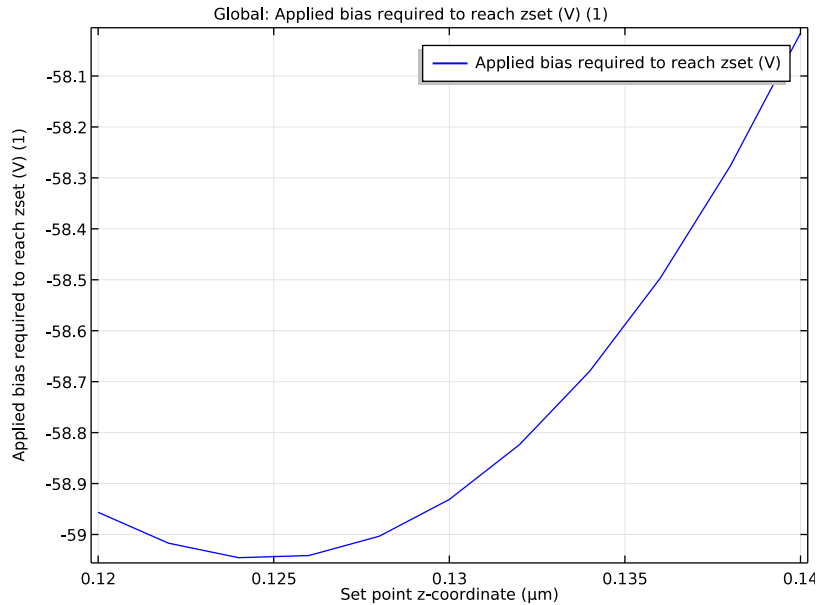

Figure 1: Voltage required to achieve a set displacement versus the target displacement. The pull-in voltage is the minimum of the plot: 59.1 V.

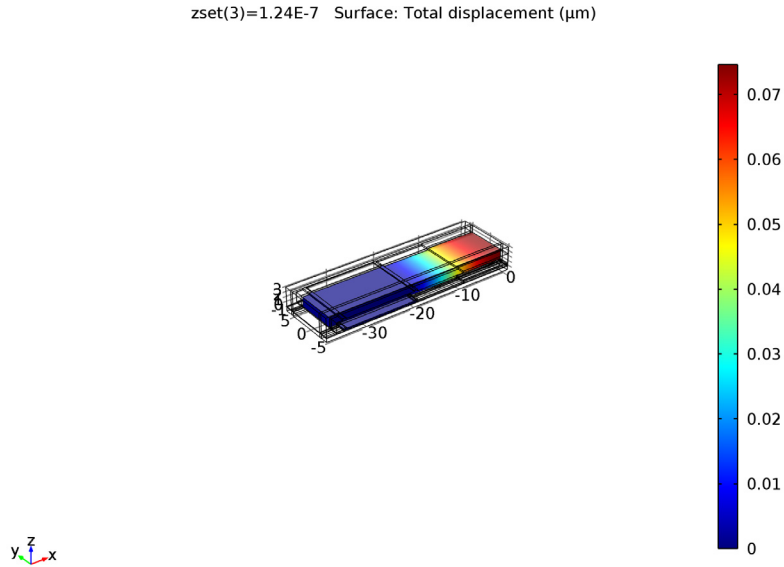zset(3)=1.24E-7   Surface: Total displacement (µm)

Figure 2: z-displacement of the resonator at pull-in. The displacement at pull-in is 75 nm. For a linear spring the displacement at pull in would be 66 nm.

Figure 2 shows the z-displacement of the resonator at the pull-in voltage. The maximum displacement at pull-in is 75 nm. This is comparable to the linear spring value of 66 nm.

Notes About the COMSOL Implementation

To compute the voltage required to generate the desired displacement of the beam, use a global equation. A common use of global equations is for computing the value of a dependent variable based on an ordinary differential equation in the dependent variable itself. However, it is also possible to couple a global equation with the other PDEs in the model as a powerful tool to solve certain kinds of inverse problems. This model uses a global equation to compute the potential applied to the drive electrode. The equation takes the form

$$z_0 = z_{set}$$

where z_0 is the z-coordinate of the midpoint of the beam's underside and z_{set} is the desired z-coordinate. COMSOL Multiphysics computes the voltage to satisfy the

constraint implied by the above equation. Note that the large difference in scale between the set-point displacement (10^{-7} m) and the applied voltages (10^2 V) means that care must be taken with the dependent variable scaling in the solver settings.

Reference

1. F.D. Bannon III, J.R. Clark and C.T.-C. Nguyen, "High-Q HF Microelectromechanical Filters," *IEEE Journal of Solid State Circuits,* vol. 35, no. 4, pp. 512–526, 2000.

Application Library path: MEMS_Module/Actuators/
biased_resonator_3d_pull_in

Modeling Instructions

Open the existing stationary study (filename: biased_resonator_3d_basic.mph).

From the **File** menu, choose **Open**.

Browse to the application's Application Library folder and double-click the file biased_resonator_3d_basic.mph.

Add a parameter to set the z-coordinate of the midpoint of the resonator.

GLOBAL DEFINITIONS

Parameters

1 In the **Model Builder** window, expand the **Global Definitions** node, then click **Parameters**.

2 In the **Settings** window for Parameters, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
zset	100[nm]	IE-7 m	Set point z-coordinate

Add an integration operator to compute the actual height of the set-point.

DEFINITIONS

Integration 1 (intop1)

1 On the **Definitions** toolbar, click **Component Couplings** and choose **Integration**.

2 In the **Settings** window for Integration, locate the **Source Selection** section.

3 From the **Geometric entity level** list, choose **Point**.

4 Select Point 217 only.

Change the drive potential to the value VdcSP - which will be solved for in a global equation.

ELECTROMECHANICS (EMI)

Electric Potential 1

1 In the **Model Builder** window, expand the **Component 1 (comp1)>Electromechanics (emi)** node, then click **Electric Potential 1**.

2 In the **Settings** window for Electric Potential, locate the **Electric Potential** section.

3 In the V_0 text field, type VdcSP.

Add a global equation to compute VdcSP.

4 In the **Model Builder** window's toolbar, click the **Show** button and select **Advanced Physics Options** in the menu.

Global Equations 1

1 On the **Physics** toolbar, click **Global** and choose **Global Equations**.

2 In the **Settings** window for Global Equations, locate the **Global Equations** section.

3 In the table, enter the following settings:

Name	f(u,ut,utt,t) (1)	Initial value (u_0) (1)	Initial value (u_t0) (1/s)	Description
VdcSP	intop1(z)-z set	0	0	Applied bias required to reach zset (V)

Set up a parametric sweep over zset.

ADD STUDY

1 On the **Home** toolbar, click **Add Study** to open the **Add Study** window.

2 Go to the **Add Study** window.

3 Find the **Studies** subsection. In the **Select study** tree, select **Preset Studies>Stationary**.

4 Click **Add Study** in the window toolbar.

5 On the **Home** toolbar, click **Add Study** to close the **Add Study** window.

STUDY 2

Step 1: Stationary

1 In the **Model Builder** window, under **Study 2** click **Step 1: Stationary**.

2 In the **Settings** window for Stationary, click to expand the **Study extensions** section.

3 Locate the **Study Extensions** section. Select the **Auxiliary sweep** check box.

4 Click **Add**.

5 In the table, enter the following settings:

Parameter name	Parameter value list	Parameter unit
zset		

6 Click **Range**.

7 In the **Range** dialog box, type 120e-9 in the **Start** text field.

8 In the **Step** text field, type 2e-9.

9 In the **Stop** text field, type 140e-9.

10 Click **Replace**.

The dependent variables require scaling correctly in order to assist the solver.

Solution 2

1 On the **Study** toolbar, click **Show Default Solver**.

2 In the **Model Builder** window, expand the **Solution 2** node, then click **Dependent Variables 1**.

3 In the **Settings** window for Dependent Variables, locate the **General** section.

4 From the **Defined by study step** list, choose **User defined**.

5 In the **Model Builder** window, expand the **Study 2>Solver Configurations>Solution 2>Dependent Variables 1** node, then click **Electric potential (comp1.V)**.

6 In the **Settings** window for Field, locate the **Scaling** section.

7 From the **Method** list, choose **Manual**.

8 In the **Scale** text field, type 100.

9 In the **Model Builder** window, under **Study 2>Solver Configurations>Solution 2>Dependent Variables 1** click **Spatial coordinates (Material) (comp1.xyz)**.

10 In the **Settings** window for Field, locate the **Scaling** section.

11 From the **Method** list, choose **Manual**.

12 In the **Scale** text field, type 1e-6.

13 In the **Model Builder** window, under **Study 2>Solver Configurations>Solution 2>Dependent Variables 1** click **Applied bias required to reach zset (V) (comp1.ODE1)**.

14 In the **Settings** window for State, locate the **Scaling** section.

15 From the **Method** list, choose **Manual**.

16 In the **Scale** text field, type 100.

The inverse problem is highly non-linear, so the solver settings need to be adjusted accordingly.

17 In the **Model Builder** window, expand the **Study 2>Solver Configurations>Solution 2>Stationary Solver 1** node.

18 Right-click **Study 2>Solver Configurations>Solution 2>Stationary Solver 1** and choose **Fully Coupled**.

19 In the **Settings** window for Fully Coupled, locate the **General** section.

20 From the **Linear solver** list, choose **Direct**.

21 Click to expand the **Method and termination** section. Locate the **Method and Termination** section. From the **Nonlinear method** list, choose **Automatic highly nonlinear (Newton)**.

22 In the **Model Builder** window, right-click **Study 2** and choose **Rename**.

23 In the **Rename Study** dialog box, type Pull In in the **New label** text field.

24 Click **OK**.

25 On the **Study** toolbar, click **Compute**.

RESULTS

Displacement (emi) 1
Determine the pull-in voltage by plotting VdcSP vs. zset.

1D Plot Group 6

1 On the **Home** toolbar, click **Add Plot Group** and choose **1D Plot Group**.

2 In the **Settings** window for 1D Plot Group, locate the **Data** section.

3 From the **Data set** list, choose **Pull In/Solution 2**.

4 On the **ID Plot Group 6** toolbar, click **Global**.

5 In the **Settings** window for Global, click **Replace Expression** in the upper-right corner of the **y-axis data** section. From the menu, choose **Component 1>Electromechanics>VdcSP - Applied bias required to reach zset (V)**.

6 Locate the **x-Axis Data** section. From the **Parameter** list, choose **Expression**.

7 In the **Expression** text field, type zset.

8 In the **Model Builder** window, right-click **ID Plot Group 6** and choose **Rename**.

9 In the **Rename ID Plot Group** dialog box, type Pull-In Plot in the **New label** text field.

I0 Click **OK**.

II On the **Pull-In Plot** toolbar, click **Plot**.

I2 Click the **Zoom Extents** button on the **Graphics** toolbar.

Compare the resulting plot with Figure 1. The pull in voltage is the minimum of the curve: 59.1 V at zset=124 nm.

Now look at the displacement at pull in. The default plot group can be used.

Displacement (emi) 1

I In the **Model Builder** window, under **Results** click **Displacement (emi) 1**.

2 In the **Settings** window for 3D Plot Group, locate the **Data** section.

3 From the **Parameter value (zset)** list, choose **1.24E-7**.

4 On the **Displacement (emi) 1** toolbar, click **Plot**.

Compare the resulting plot with Figure 2. The displacement at pull-in is 75 nm.